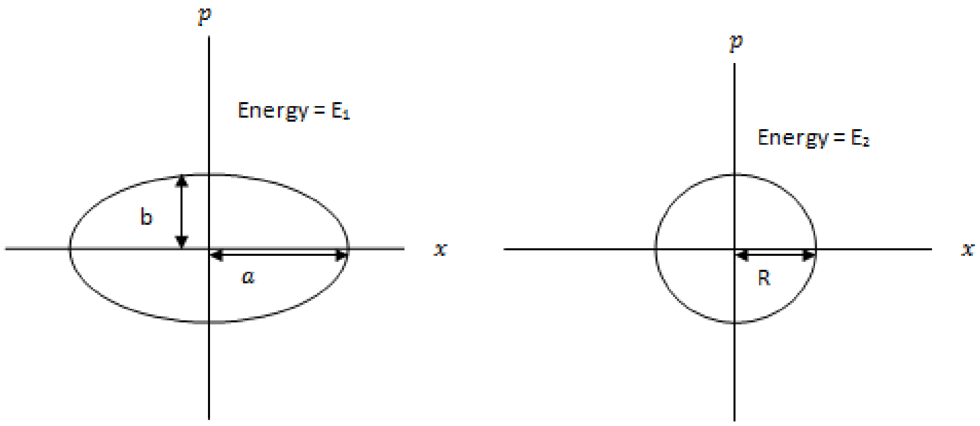
11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$ then the correct equation(s) is (are)



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_1 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Answer:

Here at $x = a, momentum\ is\ minimum\ p = 0\ and\ at\ x = 0\ momentum\ p\ is\ maximum = b$

We know that maximum momentum at $x = 0, b = m\omega_1 a\ or\ \frac{b}{a} = m\omega_1 \rightarrow (1)$

Second harmonic oscillator being circular maximum momentum = R at $x = 0$

Therefore $R = m\omega_2 R\ or\ 1 = m\omega_2 \rightarrow (2)$

Dividing equation (1) by equation (2):

$\frac{m\omega_1}{m\omega_2} = \frac{b}{a}\ or\ \frac{\omega_1}{\omega_2} = \frac{b}{a}\ \left[given\ \frac{a}{b} = n^2 or\ \frac{b}{a} = \frac{1}{n^2}\right] or\ \frac{\omega_1}{\omega_2} = \frac{1}{n^2}\ or \frac{\omega_2}{\omega_1} = n^2$ (option B is correct)

Also $E_1 = \frac{1}{2}m{\omega_1}^2 a^2\ and\ E_2 = \frac{1}{2}m{\omega_2}^2 R^2\ or\ \frac{E_1}{E_2} = \frac{{\omega_1}^2 a^2}{{\omega_2}^2 R^2} = \left(\frac{\omega_1}{\omega_2}\right)^2 \left(\frac{a}{R}\right)^2\ [\,given\ \frac{a}{R} = n]$

Therefore $\frac{E_1}{E_2} = \frac{\omega_1}{\omega_2} \times \frac{1}{n^2} \times n^2 or\ \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$ (option D is also correct)

Correct options are (B) and (D)