16. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature *T*. Assuming the gases are ideal, the correct statement(s) is(are)

(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

Answer:

$Number\ of\ moles\ of\ hydgrogen(n_1)$
$= Number\ of\ moles\ of\ Helium(n_2) = 1$
$M_1 = molecular\ weight\ of\ hydrogen = 2$

$M_2 = molecular\ weight\ of\ helium = 4$

$$M_{mix} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2} = \frac{1 \times 2 + 1 \times 4}{1 + 1} = 3$$

For Helium: Monatomic, double bond, degree of freedom=3, $\gamma_{He} = \frac{C_{pHe}}{C_{vHe}} = 1.44$

For Hydrogen : It is diatomic, single bond, degree of freedom=5,$\gamma_{H_2} = \frac{C_{pH_2}}{C_{vH_2}} = 1.66$

Putting respective values of Specific heat capacity at constant pressure and Sp. Heat capacity at constant volume

$$\gamma_{mix} = \frac{\frac{n_1C_{pH_2} + n_2C_{pHe}}{n_1 + n_2}}{\frac{n_1C_{vH_2} + n_2C_{vHe}}{n_1 + n_2}} = \frac{\frac{1 \times \frac{5}{2} + 1 \times \frac{7}{2}}{1 + 1}}{\frac{1 \times \frac{3}{2} + 1 \times \frac{5}{2}}{1 + 1}}$$

$$= \frac{12}{8} = \frac{3}{2}$$

Option (A) :

Average Energy $= \frac{1}{2}n_1f_1RT + \frac{1}{2}n_2f_2RT =$
$\frac{3}{2}RT + \frac{5}{2}RT = 4RT$
$n_1 + n_2 = 2$
Therefore Average Energy per mole$= \frac{4RT}{2} = 2RT$
[Option (A) is correct.]

Option (B)

$$V = \sqrt{\frac{\gamma RT}{M}}\ Hence\ \frac{V_{mix}}{V_{He}} = \sqrt{\frac{\gamma_{mix} \times M_{He}}{M_{mix} \times \gamma_{He}}}$$

$$= \sqrt{\frac{\frac{3}{2} \times 4}{3 \times \frac{5}{3}}} = \sqrt{\frac{6}{5}}$$ [Option B is correct.]

Option (C) & (D): Root mean square velocity

We know $V_{rms} = \sqrt{\frac{3RT}{M}}$, $\frac{V_{rmsHe}}{V_{rmsH2}} = \sqrt{\frac{M_{H2}}{M_{He}}} = \sqrt{\frac{1}{2}}$
[Option (D) is also correct]

Correct options are (A), (B) and (D)