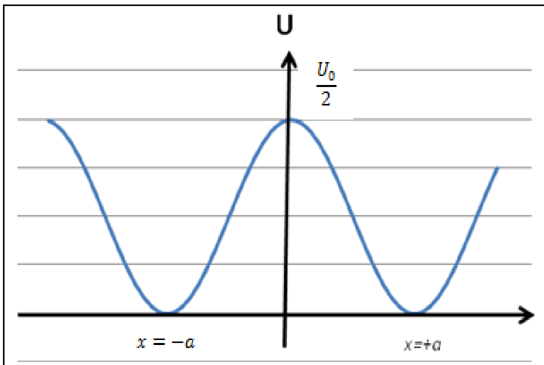
SelfStudy.in

20. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I ($a$ and $U_0$ are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

| Column I | Column II |
|---|---|
| (A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$ | (P) The force acting on the particle is zero at $x = a$. |
| (B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$ | (Q) The force acting on the particle is zero at $x = 0$. |
| (C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 exp\left[-\left(\frac{x}{a}\right)^2\right]$ | (R) The force acting on the particle is zero at $x = - a$. |
| (D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$ | (S) The particle experiences an attractive force towards x = 0 in the region $\|x\| < a$ |
| | (T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = - a$. |

**Answer**:




$U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2 \times \left(-\frac{2x}{a^2}\right)$

Energy =0 at $x = \pm a$

$\frac{dU_1}{dx}$ =$2\frac{U_0}{2}\left[\left[1-\left(\frac{x}{a}\right)^2\right]^2\right] \times \left(-\frac{2x}{a^2}\right)$

(A) We know that $F = -\frac{dU_1}{dx}$

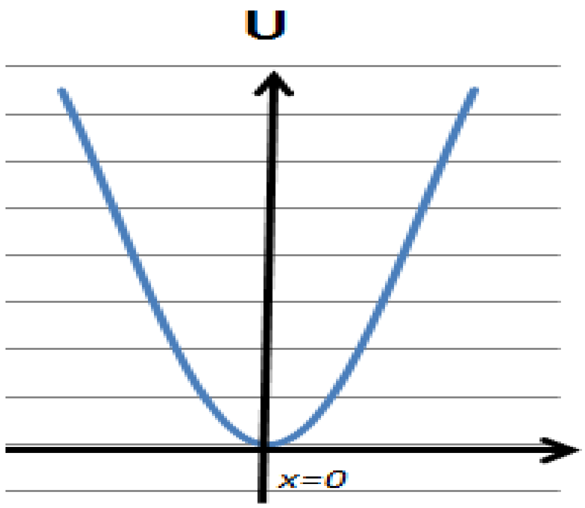
To find where F=0, clearly at $x = 0, F = 0\ also\ at\ x = a, F = 0, at\ x = -a, F$=0

Also from pictorial representation of $U_1(x)$.

$x = -a$ is position of stable equilibrium if particle has energy $\frac{U_0}{4}$ half of peak($\frac{U_0}{2}$), it can not reach to peak and can't cross hence it will oscillate.

At $x = 0$, if we release a particle it will not shift towards $x = 0$,rather it will fall to lower its energy hence force is not attractive. Option (S) is not correct.

**(A) → P, Q, R & T**




(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$ This represents a parabola with vertex at $x = 0$

$$\frac{dU_2}{dx} = 2\frac{U_0}{2}\left(\frac{x}{a}\right)$$

$F = -\frac{dU_2}{dx} = -\frac{U_0 x}{a}$ → (1)

Also $|x| < a$, Force is attractive

Force is not zero at $x = a$, Option P is wrong.
From above equation
From equation (1) $at\ x = 0, F = 0$, Option Q is correct

$F \neq 0\ at\ x = -a$, Option R is wrong.

Particle
$|x| < a$ particle will experience attractive force towards mean position i.e. $x = 0$
As potential energy will tend to lower its value. Option S is correct.

Oscillation not possible at
$x = -a\ as\ it\ is\ not\ equilibirium\ position$, as per the graph. Option T is wrong.

**(B)→Q, S**

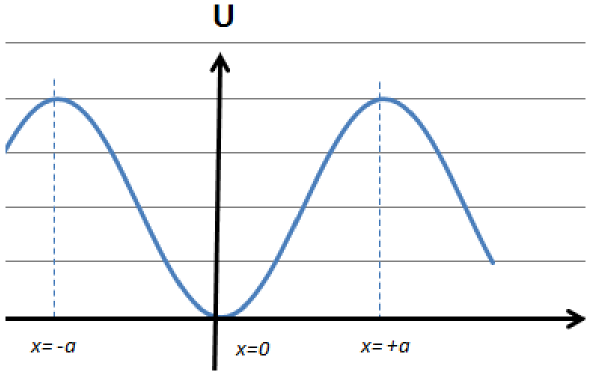
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 exp\left[-\left(\frac{x}{a}\right)^2\right]$

$$\frac{dU_3}{dx} = \left[\frac{U_0}{2}\left(\frac{x}{a}\right)^2 \times exp\left\{-\left(\frac{x}{a}\right)^2\right\} \times \left(-\frac{2x}{a^2}\right)\right] + exp\left\{-\left(\frac{x}{a}\right)^2\right\} \times 2\frac{U_0}{2}\left(\frac{x}{a^2}\right)$$

$$\frac{dU_3}{dx} = \frac{U_0}{a^2} \times x \times\ exp\left\{-\left(\frac{x}{a}\right)^2\right\}\left[1 - \left(\frac{x}{a}\right)^2\right]$$

$$F = -\frac{dU_3}{dx} = -\frac{U_0}{a^2} \times x \times\ exp\left\{-\left(\frac{x}{a}\right)^2\right\}\left[1 - \left(\frac{x}{a}\right)^2\right]$$

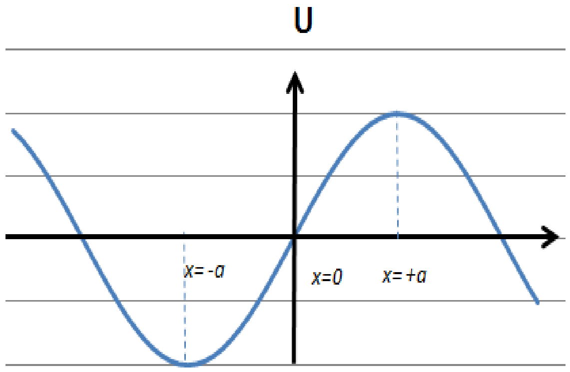
Thus $F = 0\ at\ x = 0, at\ x = +a, x = -a$




Also $if\ |x| < a\ then\ F\ is\ Negative$, $indicating\ attractive\ force\ will\ be\ experienc$

From the above energy position graph we find position $x = -a$ it is in unstable equilibrium hence oscillation at this point ruled out.

**(C)→P, Q, R & S**




$$(D)U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$$

$$\frac{dU_4}{dx} = \frac{U_0}{2}\left[\frac{1}{a} - \frac{3x^2}{3a^3}\right]$$

$$F = -\frac{dU_4}{dx} = -\frac{U_0}{2}\left[\frac{1}{a} - \frac{3x^2}{3a^3}\right]$$

Thus F=0 at $x = a\ and\ at\ x = -a$
Option (P) is correct F=0. At $x = a$
Option (Q) is wrong as at $x = 0, F \neq 0$
Option (R) is correct as F=0. At $x = -a$
Option (S) : Now there are two sections from $x = 0\ to\ x = +a$, $particle\ if\ left\ free\ will\ tend\ to\ move\ towards\ x = 0$
But $x = 0\ to\ x = -a$,lower half of graph particle will not move towards $x = 0$ if left free no attractive force.
So option (S) is wrong.
Option (T): From the position energy suggestive graph $x = -a$ is stable equilibrium position. Hence particle will oscillate. This is also correct option.
**(D)→P, R & T.**