(3) A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is ¼ th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

**Answer**: Let the bullet reaches upto a height h from the centre of earth then acceleration due to gravity at a height h

$$g_h = \frac{GM}{h^2} \rightarrow (1)$$

Acceleration due to gravity on the surface of earth $g_R = \frac{GM}{R^2} \rightarrow (2)$

Give $g_h = \frac{g_R}{4}$ $or$ $\frac{GM}{h^2} = \frac{GM}{4R^2}$ $or$ $h = 2R \rightarrow (3)$

Applying conservation of energy principle

Energy on the surface = Energy at a height h

$$(P.E + K.E)_{Surface} = \left((P.E + K.E)\right)_{at\ a\ height\ h}$$

$$-\frac{GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{r} + 0$$

$$\frac{1}{2}mv^2 = \frac{-GMm}{r} + \frac{GMm}{R}$$

$$or\ \frac{1}{2}mv^2 = \frac{-GMm}{2R} + \frac{GMm}{R}$$

$$or\ \frac{1}{2}mv^2 = \frac{GMm}{2R}\ or\ v = \sqrt{\frac{GM}{R} \rightarrow (3)}$$

$$but\ we\ know\ Escape\ Velocity\ V_e = \sqrt{\frac{2GM}{R}} = v\sqrt{2} \rightarrow (4)$$

$$Given\ V_e = v\sqrt{N} \rightarrow (5)$$

From (4) and (5) N=2.

**Answer: 2**