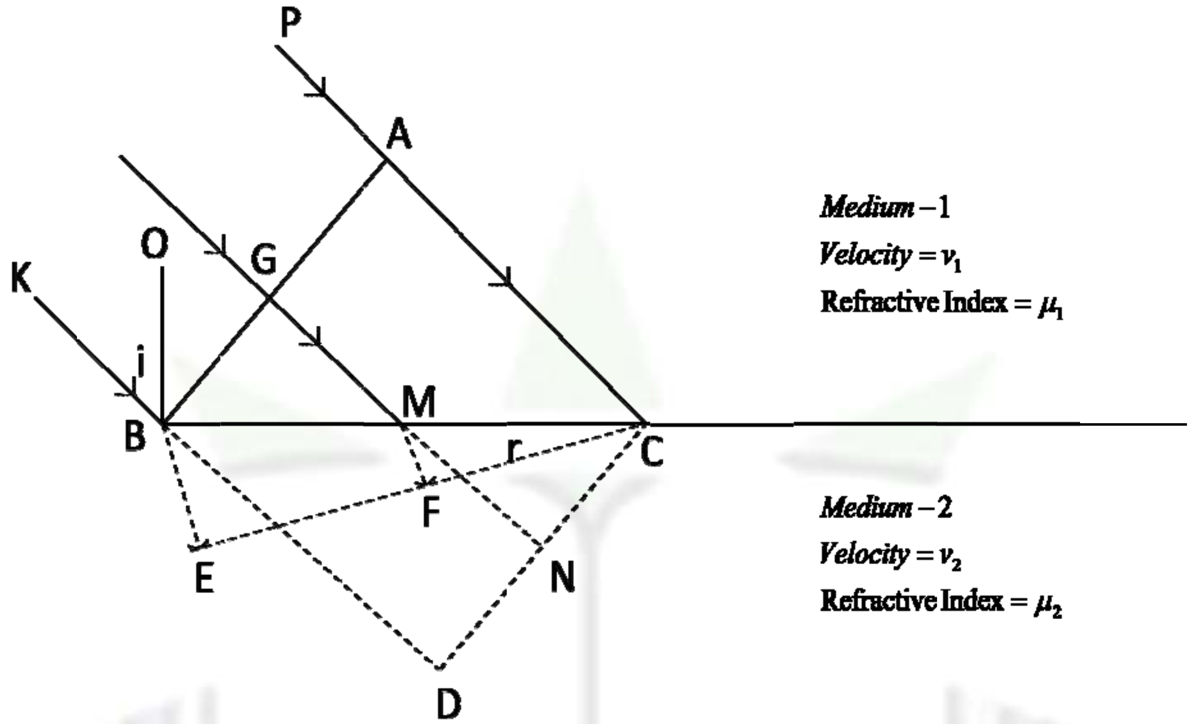
Explanation of laws of refraction by wave theory:



BC is the section of the refracting surface by the plane of the paper i.e. the out face, separating two media of refractive indices μ_1 and μ_2 ($\mu_1 < \mu_2$)

AB is the section of the incident plane wave front by the plane of the paper at an instant of time t=0

Both the refracting surface and the plane wave front are perpendicular to the plane of the paper.

v_1 & v_2 = Velocity of light in medium-1 and medium-2 respectively.

PA and KB are perpendiculars dropped on the incident wave front AB and hence they are the incident rays. BO is normal drawn at B on the refracting surface.

$$\angle KBO = i = \text{angle of incidence}$$
$$\angle KBA = 90 \quad \therefore \angle OBA = 90 - i$$
$$\angle OBC = 90 \;\; \therefore \angle ABC = 90 - (90 - i) = \text{i}$$

Hence angle of incidence can also be defined as the angle between the incident wave front and the refracting surface. Similarly the angle of refraction can also be defined as the angle between the refracted wave front and the refracting surface.

Had the two medium been same, then the rays from the point G & B would have gone up to N & D respectively during the time t in which ray from a goes up to C so that

$$AC = GN = BD = v_1 t \longrightarrow (1)$$

Hence CND would have been the position of the wave front after time t.

But since the two medium are different let the ray from B go up to E in time t. So that

$$t = \frac{AC}{v_1} = \frac{BE}{v_2} \rightarrow (2)$$

With B as center and BE as radius we imagine a sphere the section of which by the plane of the paper gives an arc of a circle. CE is drawn tangential to that arc. Then a plane through CE and perpendicular to the plane of the paper would represent the refracted wave front after time t provided we can show that ray from any other point G on the incident wave front also touches the plane through CE after time t.

To prove it, from M, MF is drawn perpendicular to CE.

$$\Delta BDC \ \& \ \Delta \mathrm{MNC} \text{ are similar } \therefore \frac{\mathrm{BD}}{\mathrm{MN}} = \frac{BC}{MC} \rightarrow (3)$$

$$\Delta BEC \ \& \ \Delta \mathrm{MFC} \text{ are similar } \therefore \frac{\mathrm{BE}}{\mathrm{MF}} = \frac{BC}{MC} \rightarrow (4)$$

$$From\ (3)\ \&\ (4)\ \frac{\mathrm{BD}}{\mathrm{MN}} = \frac{\mathrm{BE}}{\mathrm{MF}}$$

$$Putting \text{ equation } (1)\ \&\ (2)\ \frac{\mathrm{v}_1 t}{MN} = \frac{v_2 t}{MF} \text{ or } \frac{\mathrm{MF}}{\mathrm{v}_2} = \frac{MN}{v_1}$$

That is time taken to go from M to F in medium 2 and time taken to go from M to N in medium 1 is same.

Hence the time taken by the light to go from G to M + M to N is same as the time taken to go from G to M + M to F i.e. from G to F.

Hence CE is the refracted wave front after time t.

$$\therefore \angle BCE = \text{angle between the refracting surface and the refracted wavefront} = \text{angle of refraction} = \text{r (using definition)}$$

We now prove Snell's law:

$$\text{From } \Delta \text{ABC:} \quad \sin \text{i} = \frac{\text{AC}}{\text{BC}}$$

$$\text{From } \Delta \text{ABC:} \quad \sin \text{r} = \frac{\text{BE}}{\text{BC}}$$

$$\therefore \frac{\sin i}{\sin r} = \frac{AC/BC}{BE/BC} = \frac{AC}{BE}$$

Putting equation (1) and (2) : $\frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t}$

$$\frac{\sin i}{v_1} = \frac{\sin r}{v_2}$$

Multiplying both sides by v_0 where v_0 is the velocity of light in vacuum.

$$\frac{v_0}{v_1}\sin i = \frac{v_0}{v_2}\sin r$$

$\mu_1 \sin i = \mu_2 \sin r$ First law proved

From the nature of construction it is evident that the incident ray refracted ray & normal at the point of incidence all lie on the same plane i.e. on the plane of the paper.