Newton's Law Of Cooling

Newton's law of cooling: When the temperature difference between the body and the surrounding is small, the radiation loss can be given by simplified formula due to Newton.

From Stefan's law rate of loss of energy ue to radation

$$U = \sigma\left[T^4 - T_0^4\right]$$

$$U = \sigma\left(T^2 - T_0^2\right)\left(T^2 - T_0^2\right)$$

$$U = \sigma\left(T - T_0\right)\left(T + T_0\right)\left(T^2 + T_0^2\right)$$

Since $T - T_0$ is very small $T \approx T_0$

$$U = \sigma 4T_0^3\left(T - T_0\right)$$

$$U \propto \left(T - T_0\right)$$

Statement: **"Provided the temperature difference is small the amount of heat lost due to radiation per second is proportional to the temperature difference between the body and the surrounding."**

$$\frac{dQ}{dt} \propto \left(\theta - \theta_0\right)$$

Explanation: Let a body cool down by radiation loss in the surrounding

Given: θ_0=Temperature of the surrounding (constant)

m & s = mass and specific heat capacity of the body.

θ = Temperature of the body at any instant of time t

$\theta - d\theta$ = Temperature of the body at time t+dt

Heat lost by the body in time dt: $dQ = msd\theta$

$$\frac{dQ}{dt} = ms\frac{d\theta}{dt} \rightarrow (1)$$

Using Newton's law of cooling

$$\frac{dQ}{dt} = K\left(\theta - \theta_0\right) \rightarrow (2)$$

Where K is constant of proportionality which depends on two factors

(i) The surface area exposed to radiation

(ii) The nature of the radiating surface

From equation(1) and (2)

$$ms\frac{d\theta}{dt} = K\left(\theta - \theta_0\right)$$

$$\frac{d\theta}{dt} = \frac{K}{ms}\left(\theta - \theta_0\right) \rightarrow (3)$$

$$\frac{d\theta}{dt} \propto \left(\theta - \theta_0\right) \rightarrow (4)$$

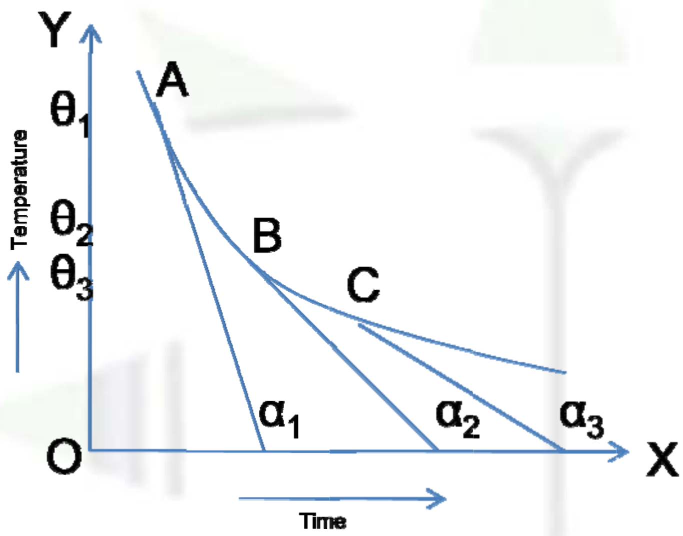
From equation (4) we find that due to radiation loss the rate of change of temperature is proportional to the temperature difference between the body and the surrounding.

Verification of Newton law of cooling:

(1) Water in a beaker is heated to a temperature about 10^0C to 15^0C above the room temperature. The hot water is then taken in a calorimeter and is allowed to cool by radiation loss.

(2) The temperature of water is recorded from the thermometer inserted into the calorimeter at intervals of every half minutes.

From this time-temperature record a cooling curve is plotted.



Few points A,B,C… are chosen on the cooling curve corresponding to the temperature $\theta_1, \theta_2, \theta_3$ respectively. Tangents drawn to the curve at the chosen points make angle α_1 , α_2 , α_3 with the time axis respectively.

Then tanα will give the slope of the curve at that point i.e.

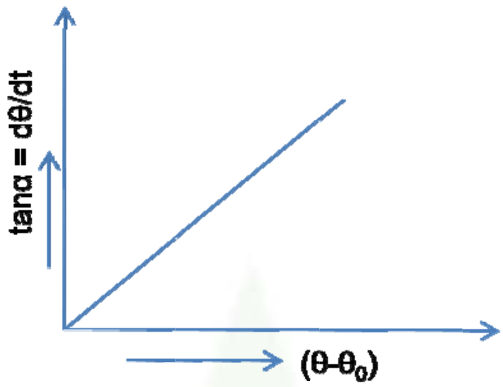
$$\tan\alpha = \left(\frac{d\theta}{dt}\right)_{\theta}$$

$$\tan\alpha_1 = \left(\frac{d\theta}{dt}\right)_{\theta=\theta_1}$$

$$\tan\alpha_2 = \left(\frac{d\theta}{dt}\right)_{\theta=\theta_2}$$

$$\tan\alpha_3 = \left(\frac{d\theta}{dt}\right)_{\theta=\theta_3}$$

Thus we get set of values $\tan\alpha_1$, $\tan\alpha_2$, $\tan\alpha_3$ corresponding to temperature θ_1, θ_2, θ_3



A graph is now plotted keeping tanα = dθ/dt along Y axis and $(\theta-\theta_0)$ along X axis, the graph is found to be a straight line which proves that

$$\frac{d\theta}{dt} \propto (\theta-\theta_0)$$

Newton's law of cooling is verified.

Experimental determination of specific heat of a liquid by Newton’s law of cooling:

(1) Two copper calorimeter having nearly equal diameter are taken and weighed.

(2) Equal volume of the experimental liquid and water are taken in two test tubes and are heated to same temperature by immersing the test tubes in hot water taken in a beaker.

(3) The contents of the test tube are transferred separately into the two calorimeters and the temperature of both the liquids is recorded at interval of every half minute.

(4) The calorimeters are weighed again to get the mass of the experimental liquid and water.

Given :

$\mathrm{m_1}$ & $\mathrm{m_2}$ = mass of the calorimeter and the experimental liquid taken in that calorimeter respectively.

$\mathrm{m_1'}$ & $\mathrm{m_2'}$ = mass of the calorimeter and the water taken in that calorimeter respectively.

s, s_1, s_2 = specific heat of the calorimeter, water & the experimental liquid respectively.

Using Newton's law of cooling :

(i) For loss of heat due to radiation by the experimental liquid

$$(\mathrm{m_1}s + \mathrm{m_2}s_2)\left(\frac{d\theta}{dt}\right)_l = K(\theta - \theta_0) \rightarrow (1)$$

where $\left(\frac{d\theta}{dt}\right)_l$ = Rate of fall of temperature of the experimental liquid at the temperature θ.

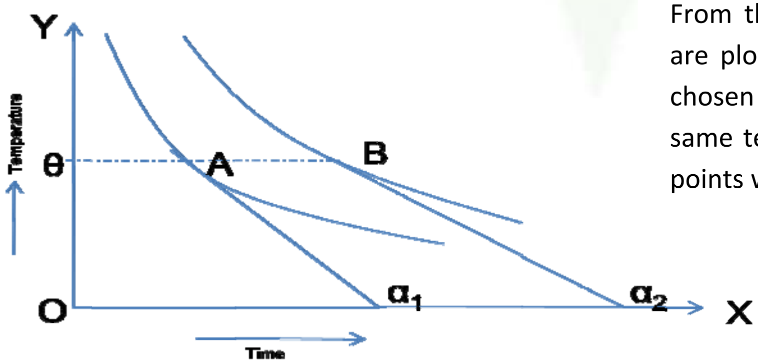
(ii) For the radiation loss by water

$$(\mathrm{m_1'}s + \mathrm{m_2'}s_1)\left(\frac{d\theta}{dt}\right)_w = K(\theta - \theta_0) \rightarrow (2)$$

where $\left(\frac{d\theta}{dt}\right)_w$ = Rate of fall of temperature of the water at the temperature θ.

From equation (1) and (2): $(\mathrm{m_1}s + \mathrm{m_2}s_2)\left(\frac{d\theta}{dt}\right)_l = (\mathrm{m_1'}s + \mathrm{m_2'}s_1)\left(\frac{d\theta}{dt}\right)_w \rightarrow (3)$

The constant K is taken same because the nature of the radiating surface and the surface area exposed to radiation (volume of both the liquids are same) are same in both.



From the time temperature record cooling curves are plotted for both the liquids. A & B are points chosen on the two curves corresponding to the same temperature θ. Tangents are drawn at those points which make angles α_1 , α_2 with the time axis.

$$\tan\alpha_1 = \left(\frac{d\theta}{dt}\right)_{w,\theta}, \tan\alpha_2 = \left(\frac{d\theta}{dt}\right)_{l,\theta}$$

Putting the values in equation(3)

$$(\mathrm{m_1}s + m_2 s_2)\tan\alpha_2 = (m_1's + m_2's_1)\tan\alpha_1$$

s_2 can be calculated because all other quantities are known.