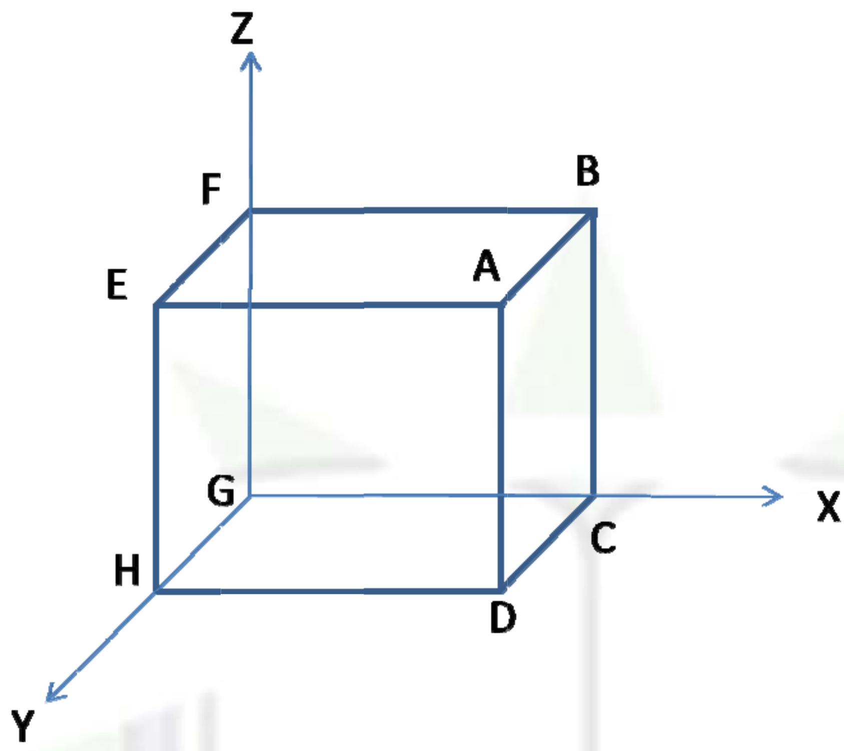
Pressure exerted by a perfect gas:



Let us consider a perfect gas enclosed in cube the walls of which are perfectly elastic. The three edges of the cube are taken to be X, Y & Z axes.

Given:

m = mass of each molecule of the gas

n = the total number of molecules of the gas

Let C1, C2, C3, Cn be the velocities of molecules of the gas.

Let us consider a molecule having a velocity $\vec{C}$.

Let u, v, w be the components of this velocity vector $\vec{C}$ along X, Y and Z axes respectively.

$$\therefore \left|\vec{C}\right| = \sqrt{u^2 + v^2 + w^2}$$

$$C^2 = \left(u^2 + v^2 + w^2\right) \rightarrow (1)$$

Let us first consider the collision of that molecule with the walls ABCD and EFGH perpendicular to X axis. The molecule moves with velocity U along X axis , strikes the wall ABCD and since the collision is perfectly elastic the molecule rebounds with same speed U along -X axis, goes and strikes the wall EFGH, rebounds with velocity U and so on.

The momentum of the molecule before and after the collision with the wall ABCD are mu and –mu respectively.

Therefore the change in momentum of the molecule due to collision with the wall ABCD

= mu – (-mu) = 2mu

Between two successive collisions with the wall ABCD the molecule covers a distance 2l with a speed u and takes time 2l/u.

Thus in every 2l/u second molecule takes 1 collision in every 1 second molecule takes 1/(2l/u) = u/2l collision.

Hence the change in momentum of the molecule in 1 sec due to collision with the wall ABCD = change in momentum in 1 collision x no. of collision in 1 sec

= 2mu x u/2l = mu^2/l

According to Newton's second law the rate of change of momentum is impressed force.

Therefore the force exerted by a molecule due to the motion along X-axis = mu^2/l

Therefore the total force exerted by all the molecules due to their motion along X-axis

$$= \sum \frac{mu^2}{l}$$

The pressure exerted by all the gas molecules due to their motion along X axis

$$P_x = \frac{F}{A} = \frac{1}{l^2}\frac{m}{l}\sum u^2$$

$$P_x = \frac{m}{l^3}\sum u^2$$

But $l^3 = V =$ Volume of the gas

$$\therefore P_x = \frac{m}{V}\sum u^2 \rightarrow (2)$$

Similarly by considering the motion of the molecules along Y and Z axes we get

$$P_y = \frac{m}{V}\sum v^2 \rightarrow (3)$$

$$P_z = \frac{m}{V}\sum w^2 \rightarrow (4)$$

Since the pressure of the gas inside a container is same in all directions

$$\therefore P_x + P_y + P_z = P + P + P = 3P$$

$$3P = \frac{m}{V}\sum u^2 + \frac{m}{V}\sum v^2 + \frac{m}{V}\sum w^2$$

$$P = \frac{m}{3V}\left[\sum u^2 + \sum v^2 + \sum w^2\right]$$

$$P = \frac{m}{3V}\left[\left(u_1^2 + u_2^2 + u_3^2 + ... + u_n^2\right) + \left(v_1^2 + v_2^2 + v_3^2 + ... + v_n^2\right) + \left(w_1^2 + w_2^2 + w_3^2 + ... + w_n^2\right)\right]$$

$$P = \frac{m}{3V}\left[\left(u_1^2 + v_1^2 + w_1^2\right) + \left(u_2^2 + v_2^2 + w_2^2\right) + \left(u_3^2 + v_3^2 + w_3^2\right) + ... + \left(u_n^2 + v_n^2 + w_n^2\right)\right]$$

$$P = \frac{m}{3V}\left[c_1^2 + c_2^2 + c_3^2 + ... + c_n^2\right]$$

$$P = \frac{mn}{3V}\left[\frac{c_1^2 + c_2^2 + c_3^2 + ... + c_n^2}{n}\right] \rightarrow (5)$$

$$\sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + ... + c_n^2}{n}} = \vec{C} = \text{Root mean square velocity} \rightarrow (6)$$

Putting (6) in (5)

$$P = \frac{1}{3}\frac{mn\vec{C}2}{V} \rightarrow (7)$$

Equation(7) gives the pressure exerted by a perfect gas.

mn = mass of the gas $\therefore \frac{mn}{V} = \rho$ = density of the gas

$$\therefore P = \frac{1}{3}\rho\vec{C}2 \rightarrow (8)$$

Deductions from Kinetic Theory:

(1) **Boyle's law from kinetic theory** : From kinetic theory pressure exerted by a perfect gas is

$$P = \frac{1}{3}\frac{mn}{V}\overline{C}^2$$

$$PV = \frac{1}{3}mn\overline{C}^2 \rightarrow (1)$$

For a given mass of gas mn = constant

We also know that the r.m.s velocity of the molecules of a given as depends only on temperature i.e. $\overline{C} \propto \sqrt{T}$

If the temperature of the gas is kept constant then $\overline{C}$ is also constant.

From equation(1) for a given mass of a gas at constnat temperature the R.M.S velocity is constant i.e.PV = constant (Boyles law)

(2) **Kinetic interpretation of temperature**: From kinetic theory pressure exerted by a perfect gas

$PV = \frac{1}{3} mn\overline{C}^2 \rightarrow (1)$

From equation of state

$PV = \alpha RT \rightarrow (2)$

Where α = no. of molecules of the gas = $\frac{\text{Mass of the gas}}{\text{Molecular weight}} = \frac{mn}{M}$

From equation (1) and (2)

$\frac{1}{3} mn\overline{C}^2 = \alpha RT$

$\frac{1}{3} mn\overline{C}^2 = \frac{mn}{M} RT \rightarrow (3)$

$\overline{C} = \sqrt{\frac{3R}{M}} \sqrt{T} \rightarrow (4)$

Since for a given gas M and R are constant

$\overline{C} \propto \sqrt{T}$

Thus for a given gas the R.M.S velocity is proportional to the square root of absolute temperature of the gas

From equation (3)

$\frac{1}{3} mn\overline{C}^2 = \frac{mn}{M} RT$

$M = m \times N$ where N = Avogadro's no.

$\frac{1}{3} mn\overline{C}^2 = \frac{mn}{m \times N} RT$

$K = \frac{R}{N} = Boltzman's$ constant

$\frac{1}{2} mn\overline{C}^2 = \frac{3}{2} KT \rightarrow (5)$

Mean kinetic energy per molecule of the gas = $\frac{3}{2} KT$

If T = 0, Mean K.E per molecule of the gas = 0 thus absolute zero can also be defined as the temperature at which the mean kinetic energy per molecule of the gas becomes zero.

(3) **Avogadro's hypothesis from kinetic theory**: let us consider two different gases say Gas1 and Gas2

m_1 and m_2 = mass of each molecule of gas1 and gas2

n_1 and n_2 = total no of molecules of gas1 and gas2

$\bar{c}_1$ and $\bar{c}_2$ = R.M.S velocity of molecules of gas1 and gas2

P_1 and P_2 = Pressure of gas1 and gas2

V_1 and V_2 = Volume of gas1 and gas2

T_1 and T_2 = Temperature of gas1 and gas2

Using the kinetic theory the pressure exerted by gas1:

$$P_1 = \frac{1}{3}\frac{m_1 n_1}{V_1}\bar{c}_1^{\,2}$$

$$P_1 V_1 = \frac{1}{3} m_1 n_1 \bar{c}_1^{\,2} \rightarrow (1)$$

The pressure exerted by gas 2

$$P_2 V_2 = \frac{1}{3} m_2 n_2 \bar{c}_2^{\,2} \rightarrow (2)$$

If $P_1 = P_2$ and $V_1 = V_2$ then $P_1 V_1 = P_2 V_2$

From equation (1) and (2) we get

$$\frac{1}{3} m_1 n_1 \bar{c}_1^{\,2} = \frac{1}{3} m_2 n_2 \bar{c}_2^{\,2}$$

$$m_1 n_1 \bar{c}_1^{\,2} = m_2 n_2 \bar{c}_2^{\,2} \rightarrow (3)$$

Using kinetic interpretation of gas1

The mean K.E per molecule of the gas1 $= \frac{1}{2} m_1 \bar{c}_1^{\,2} = \frac{3}{2} K T_1 \rightarrow (4)$

The mean K.E per molecule of gas2 $= \frac{1}{2} m_2 \bar{c}_2^{\,2} = \frac{3}{2} K T_2 \rightarrow (5)$

If $T_1 = T_2$ then from equation (4) and (5) $\frac{1}{2} m_1 \bar{c}_1^{\,2} = \frac{1}{2} m_2 \bar{c}_2^{\,2}$

Dividing equation (3) by equation (6)

$$\frac{m_1 n_1 \bar{c}_1^{\,2}}{\frac{1}{2} m_1 \bar{c}_1^{\,2}} = \frac{m_2 n_2 \bar{c}_2^{\,2}}{\frac{1}{2} m_2 \bar{c}_2^{\,2}}$$

$$\therefore n_1 = n_2$$

Thus at equal temperature and pressure equal volume of two different gases contain equal number of molecules - Avogradro's hypothesis.