Pressure exerted by a perfect gas:

Let us consider a perfect gas enclosed in cube the walls of which are perfectly elastic. The three edges of the cube are taken to be X, Y & Z axes.

Given:

\[ m = \text{mass of each molecule of the gas} \]
\[ n = \text{the total number of molecules of the gas} \]

Let \( C_1, C_2, C_3, C_n \) be the velocities of molecules of the gas.

Let us consider a molecule having a velocity \( \vec{C} \).

Let \( u, v, w \) be the components of this velocity vector \( \vec{C} \) along X, Y and Z axes respectively.

\[
\begin{align*}
\vec{C} &= \sqrt{u^2 + v^2 + w^2} \\
C^2 &= (u^2 + v^2 + w^2) \rightarrow (1)
\end{align*}
\]

Let us first consider the collision of that molecule with the walls ABCD and EFGH perpendicular to X axis. The molecule moves with velocity \( U \) along X axis, strikes the wall ABCD and since the collision is perfectly elastic the molecule rebounds with same speed \( U \) along -X axis, goes and strikes the wall EFGH, rebounds with velocity \( U \) and so on.
The momentum of the molecule before and after the collision with the wall ABCD are $\mu$ and $-\mu$ respectively.

Therefore the change in momentum of the molecule due to collision with the wall ABCD

$$= \mu - (-\mu) = 2\mu$$

Between two successive collisions with the wall ABCD the molecule covers a distance $2l$ with a speed $u$ and takes time $2l/u$.

Thus in every $2l/u$ second molecule takes 1 collision in every 1 second molecule takes $1/(2l/u) = u/2l$ collision.

Hence the change in momentum of the molecule in 1 sec due to collision with the wall ABCD = change in momentum in 1 collision x no. of collision in 1 sec

$$= 2\mu \times \frac{u}{2l} = \mu u^2 / l$$

According to Newton’s second law the rate of change of momentum is impressed force.

Therefore the force exerted by a molecule due to the motion along X-axis = $\mu u^2 / l$

Therefore the total force exerted by all the molecules due to their motion along X-axis

$$= \sum \frac{\mu u^2}{l}$$

The pressure exerted by all the gas molecules due to their motion along X axis

$$P_x = \frac{F}{A} = \frac{1}{l^2} \frac{m}{l} \sum u^2$$

$$P_x = \frac{m}{l^3} \sum u^2$$

But $l^3 = V = \text{Volume of the gas}$

$$\therefore P_x = \frac{m}{V} \sum u^2 \rightarrow (2)$$
 Kinetic Theory Of Gas-Pressure Exerted By A Gas

Similarly by considering the motion of the molecules along Y and Z axes we get

\[ P_y = \frac{m}{V} \sum v^2 \rightarrow (3) \]

\[ P_z = \frac{m}{V} \sum w^2 \rightarrow (4) \]

Since the pressure of the gas inside a container is same in all directions

\[ \therefore P_x + P_y + P_z = P + P + P = 3P \]

\[ 3P = \frac{m}{V} \sum u^2 + \frac{m}{V} \sum v^2 + \frac{m}{V} \sum w^2 \]

\[ P = \frac{m}{3V} \left[ \sum u^2 + \sum v^2 + \sum w^2 \right] \]

\[ P = \frac{m}{3V} \left[ \left( u_1^2 + u_2^2 + u_3^2 + \ldots \ldots + u_n^2 \right) + \left( v_1^2 + v_2^2 + v_3^2 + \ldots \ldots + v_n^2 \right) + \left( w_1^2 + w_2^2 + w_3^2 + \ldots \ldots + w_n^2 \right) \right] \]

\[ P = \frac{m}{3V} \left[ \left( u_1^2 + v_1^2 + w_1^2 \right) + \left( u_2^2 + v_2^2 + w_2^2 \right) + \left( u_3^2 + v_3^2 + w_3^2 \right) + \ldots \ldots + \left( u_n^2 + v_n^2 + w_n^2 \right) \right] \]

\[ P = \frac{mn}{3V} \left[ \frac{c_1^2 + c_2^2 + c_3^2 + \ldots \ldots + c_n^2}{n} \right] \rightarrow (5) \]

\[ \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + \ldots \ldots + c_n^2}{n}} = C = \text{Root mean square velocity} \rightarrow (6) \]

Putting (6) in (5)

\[ P = \frac{1}{3} \frac{mn}{V} C^2 \rightarrow (7) \]

Equation(7) gives the pressure exerted by a perfect gas.

\[ mn = \text{mass of the gas} : \frac{mn}{V} = \rho = \text{density of the gas} \]

\[ \therefore P = \frac{1}{3} \rho C^2 \rightarrow (8) \]
Deductions from Kinetic Theory:

(1) **Boyle’s law from kinetic theory** : From kinetic theory pressure exerted by a perfect gas is

\[ P = \frac{1}{3} \frac{mn \overline{C}^2}{V} \]

\[ PV = \frac{1}{3} mn \overline{C}^2 \rightarrow (1) \]

For a given mass of gas \( mn = \text{constant} \)

We also know that the r.m.s velocity of the molecules of a given gas depends only on temperature i.e. \( \overline{C} \propto \sqrt{T} \)

If the temperature of the gas is kept constant then \( \overline{C} \) is also constant.

From equation (1) for a given mass of a gas at constant temperature the R.M.S velocity is constant i.e. \( PV = \text{constant} \) (Boyle’s law)
Kinetic Theory Of Gas-Pressure Exerted By A Gas

(2) **Kinetic interpretation of temperature**: From kinetic theory pressure exerted by a perfect gas

\[ PV = \frac{1}{3} mn\bar{C}^2 \rightarrow (1) \]

From equation of state

\[ PV = dRT \rightarrow (2) \]

Where \( \bar{C} \) is no. of molecule of gas

\[ \text{Mass of gas} = \frac{mn}{M} \]

From equation (1) and (2)

\[ \frac{1}{3} mn\bar{C}^2 = dRT \]

\[ \frac{1}{3} mn\bar{C}^2 = \frac{mn}{M} RT \rightarrow (3) \]

\[ \bar{C} = \sqrt[\frac{3R}{M}} \sqrt{T} \rightarrow (4) \]

Since for a given gas M and R are constant

\[ \bar{C} \propto \sqrt{T} \]

Thus for a given gas the R.M.S. velocity is proportional to the square root of absolute temperature of the gas.

From equation (3)

\[ \frac{1}{3} mn\bar{C}^2 = \frac{mn}{M} RT \]

\[ M = m \times N \quad \text{where} \ N = \text{Avogadro's no.} \]

\[ \frac{1}{3} mn\bar{C}^2 = \frac{mn}{m \times N} RT \]

\[ K = \frac{R}{N} = \text{Boltzmann's constant} \]

\[ \frac{1}{2} mn\bar{C}^2 = \frac{3}{2} KT \rightarrow (5) \]

Mean kinetic energy per molecule of gas \( = \frac{3}{2} KT \)

If \( T = 0 \), Mean K.E per molecule of gas \( = 0 \) thus absolute zero can also be defined as the temperature at which the mean kinetic energy per molecule of the gas becomes zero.
(3) **Avogadro's hypothesis from kinetic theory**: Let us consider two different gases say Gas 1 and Gas 2.

- \( m_1 \) and \( m_2 \) = mass of each molecule of gas 1 and gas 2
- \( n_1 \) and \( n_2 \) = total no of molecules of gas 1 and gas 2
- \( c_1 \) and \( c_2 \) = R.M.S velocity of molecules of gas 1 and gas 2
- \( P_1 \) and \( P_2 \) = Pressure of gas 1 and gas 2
- \( V_1 \) and \( V_2 \) = Volume of gas 1 and gas 2
- \( T_1 \) and \( T_2 \) = Temperature of gas 1 and gas 2

Using the kinetic theory, the pressure exerted by gas 1:

\[
P_1 = \frac{1}{3} \frac{m_1 n_1 c_1^2}{V_1}
\]

\[
P_1 V_1 = \frac{1}{3} m_1 n_1 c_1^2 \rightarrow (1)
\]

The pressure exerted by gas 2:

\[
P_2 V_2 = \frac{1}{3} m_2 n_2 c_2^2 \rightarrow (2)
\]

If \( P_1 = P_2 \) and \( V_1 = V_2 \), then \( P_1 V_1 = P_2 V_2 \)

From equation (1) and (2) we get:

\[
\frac{1}{3} m_1 n_1 c_1^2 = \frac{1}{3} m_2 n_2 c_2^2
\]

\[
m_1 n_1 c_1^2 = m_2 n_2 c_2^2 \rightarrow (3)
\]

Using kinetic interpretation of gas 1:

The mean K.E. per molecule of the gas 1:

\[
\frac{1}{2} m_1 c_1^2 = \frac{3}{2} K T_1 \rightarrow (4)
\]

The mean K.E. per molecule of gas 2:

\[
\frac{1}{2} m_2 c_2^2 = \frac{3}{2} K T_2 \rightarrow (5)
\]

If \( T_1 = T_2 \), then from equation (4) and (5):

\[
\frac{1}{2} m_1 c_1^2 = \frac{1}{2} m_2 c_2^2
\]

Dividing equation (3) by equation (6):

\[
\frac{m_1 n_1 c_1^2}{\frac{1}{2} m_1 c_1^2} = \frac{m_2 n_2 c_2^2}{\frac{1}{2} m_2 c_2^2}
\]

\[
\therefore n_1 = n_2
\]

Thus, at equal temperature and pressure, equal volume of two different gases contain equal number of molecules - Avogadro's hypothesis.