The root mean square value of A.C (RMS) : Since the value of emf (or current) in an A.C cycle continuously changes with time hence we must represent the emf (or current) in an A.C cycle by suitable mean value, over a complete cycle. If we take arithmetic mean since the total value of emf (or current) in the two half cycles are equal and opposite hence the total over a complete cycle is zero. Hence to get a suitable mean value we square all the instantaneous values where by all become positive and then are added and the mean is taken. Finally to get rid of square the square root is taken. This value is known as root mean square value.

$\text{Let } e = e_p \sin\omega t \rightarrow (1)$

$\text{The instataneous emf i.e. the value of emf at an instant of time } t$

$$e_{rms} = \sqrt{\frac{1}{T}\int_0^T e^2 dt}$$

$$e_{rms}^2 = \frac{1}{T}\int_0^T e^2 dt$$

$$e_{rms}^2 = \frac{1}{T}\int_0^T e_p^2 \sin^2\omega t dt$$

$$e_{rms}^2 = \frac{e_p^2}{2T}\int_0^T 2\sin^2\omega t dt$$

$$e_{rms}^2 = \frac{e_p^2}{2T}\int_0^T (1-\cos 2\omega t) dt$$

$$e_{rms}^2 = \frac{e_p^2}{2T}\left[\int_0^T dt - \int_0^T (\cos 2\omega t) dt\right]$$

$$e_{rms}^2 = \frac{e_p^2}{2T}\left[T - \frac{1}{2\omega}(\sin 2\omega T - \sin 2\omega.0)\right]$$

$$But\ \omega = \frac{2\pi}{T} \therefore \sin 2\omega T = \sin 4\pi = 0$$

$$e_{rms}^2 = \frac{e_p^2}{2T}\left[T - \frac{1}{2\omega}(0-0)\right]$$

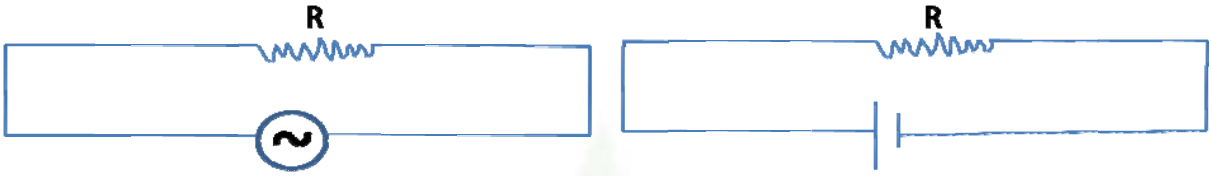
$$e_{rms}^2 = \frac{e_p^2}{2}$$

$$e_{rms} = \sqrt{\frac{e_p^2}{2}}$$

$$e_{rms} = \frac{e_p}{\sqrt{2}}$$

$$i_{rms} = \frac{i_p}{\sqrt{2}}$$

Physical significance of rms value:



The A.C source is connected across a resistance R and let H joules of heat is produced per sec. When a D.C source is connected across the same resistance a constant current say i flow through the resistance and let again H joules of heat is produced per second. Then this direct value of current is said to be rms value of current of the given A.C.

Thus rms value of current in an A.C circuit can be defined as that steady value of current which would produce the same heating effect as the given A.C. The ammeters and voltmeters used for measurement of direct current and p.d cannot be used for measuring alternating current and p.d.

A.C meters (ammeter & voltmeter) are based on heating effect of current ($H=i^2 Rt$) because heating effect of current is independent of the direction of current. The A.C meters are known as hot wire instruments. Reading of A.C meters gives the rms value.