Refraction at double spherical surface:

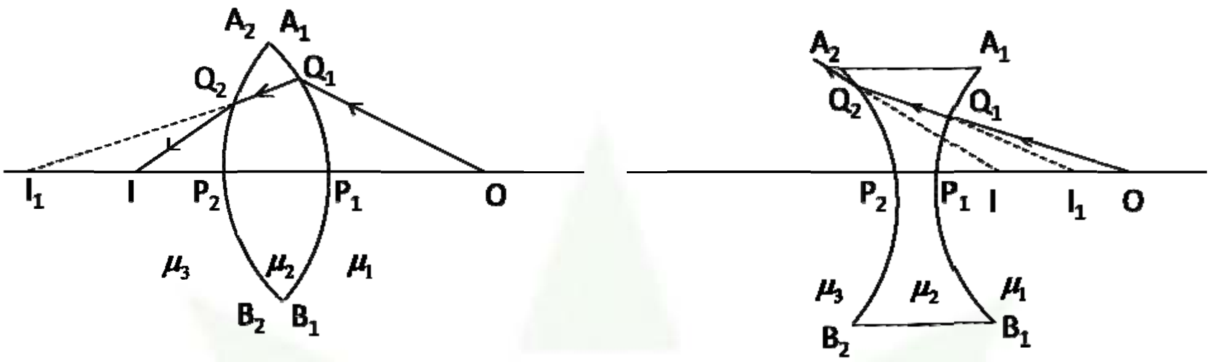


Fig-1 Fig-2

A_1B_1 & A_2B_2 are two spherically curved surfaces separating media of refractive indices μ_2 , μ_1 and μ_3 , μ_2 respectively. The medium bounded by the curved surface is denser than both the surrounding media i.e. ($\mu_1 < \mu_2 > \mu_3$). In figure-1 the concave faces bound the denser medium and in figure-2 the convex faces bound the denser medium.

Ray diagram: A point object O is kept on the common principal axis in the medium of refractive index μ_1. A ray from O is incident at any point Q_1 on the curved surface A_1B_1 & after refraction since it is going from rarer to denser medium it bends towards the normal and is refracted along Q_1Q_2. Another ray from O incident along the principal axis OP_1 being incident normally passes without any deviation along P_1P_2 , these two refracted rays Q_1Q_2 and P_1P_2 meet at I_1 directly in figure-1 & on producing back in figure-2. Hence I_1 is the image of the object O formed by refraction at the curved surface A_1B_1.

For the curved surface A_2B_2, Q_1Q_2 & P_1P_2 are incident rays since they appear to converge (in figure-1) or diverge (in figure-2) from the point I_1, hence I_1 serves as object for A_2B_2. Since Q_1Q_2 goes from denser to rarer medium bends away from the normal where as P_1P_2 passes without any deviation & meet at I. Hence I is the image of the object O formed by the double spherical surface.

Thus I is the final image of the object O formed by the double spherical surface.

Calculation: We know that for refraction at a single spherical surface with object in the rarer medium

$$\frac{\mu_o}{u} + \frac{\mu_i}{v} = \frac{\mu_i - \mu_o}{r} \rightarrow (1)$$

For refraction at curved surface A_1B_1

Refractive index of the object medium $\mu_o = \mu_1$

Refractive Index of the image medium $\mu_i = \mu_2$

Object distance $P_1O=u$

Image distance=$P_1I=v_1$

Radius of curvature =r_1

Using equation (1)

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r_1} \rightarrow (2)$$

For refraction at curved surface A_2B_2

Refractive index of the object medium $\mu_o=\mu_2$

Refractive index of the image medium $\mu_i=\mu_3$

Since μ2> μ3 therefore μo> μi , since the object is in denser medium we have to Consider sign correction before using the equation(1).

Figure- 1 : Object distance = P_2I_1=u'(say)=Negative

Image distance = P_2I=v=Positive

Radius of curvature =r_2= Negative

Using equation (1)

$$\frac{\mu_2}{-u'}+\frac{\mu_3}{v}=\frac{\mu_3-\mu_2}{-r_2}$$

$$\frac{\mu_2}{-u'}+\frac{\mu_3}{v}=\frac{\mu_2-\mu_3}{r_2}\rightarrow(3)$$

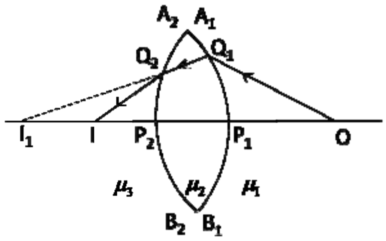


Fig-1

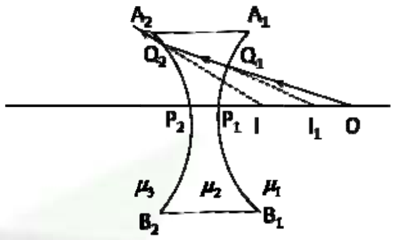


Fig-2

For figure (2): Object distance $P_2I_1 = u' = Positive$

Image distance $P_2I = v = Negative$

Radius of curvature $r_2 = Positive$

Using equation (1)

$$\frac{\mu_2}{u'}+\frac{\mu_3}{-v}=\frac{\mu_3-\mu_2}{r_2}$$

$$-\frac{\mu_2}{u'}+\frac{\mu_3}{v}=\frac{\mu_2-\mu_3}{r_2}\rightarrow(4)$$

From equation (3) & (4) we find that for both the figures equation are same, adding equation (2) & (3) or (4)

$$\frac{\mu_1}{u}+\frac{\mu_2}{v_1}-\frac{\mu_2}{u'}+\frac{\mu_3}{v}=\frac{\mu_2-\mu_1}{r_1}+\frac{\mu_2-\mu_3}{r_2}$$

$$v_1 = P_1I = P_2I \pm P_1P_2 = u' \pm t$$

$P_1P_2 = t =$ thickness of the lens for thin lens $v_1 = u'$

$$\frac{\mu_1}{u}+\frac{\mu_3}{v}=\frac{\mu_2-\mu_1}{r_1}+\frac{\mu_2-\mu_3}{r_2}\rightarrow(5)$$

Equation (5) gives the relation between the object distance image distance and radius of curvature for refraction through a lens, when the two surrounding media are different.

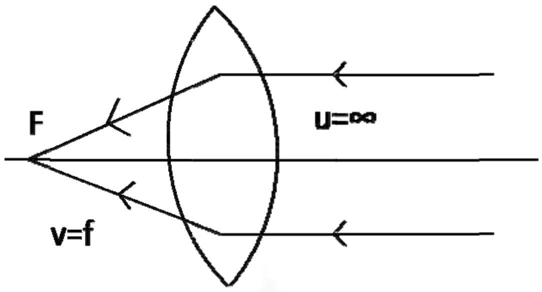
Special Case I: Let both the media surrounding the lens be same $\mu_1 = \mu_3$

$$\frac{\mu_1}{u}+\frac{\mu_1}{v}=\frac{\mu_2-\mu_1}{r_1}+\frac{\mu_2-\mu_1}{r_2}$$

$$\frac{1}{u}+\frac{1}{v}=\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{1}{u}+\frac{1}{v}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\rightarrow(6)$$

If rays are incident parallel to the principal axis ($u = \infty$), then the point on which image is formed is said to be the FOCUS. The distance of the focus from the optical center is known as focal length (f).



From equation (6)

$$\frac{1}{\infty}+\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$$

$$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\rightarrow(7)$$

Equation (7) gives the focal length of lens when the surrounding medium is other than air.