Refraction at single spherical surface:

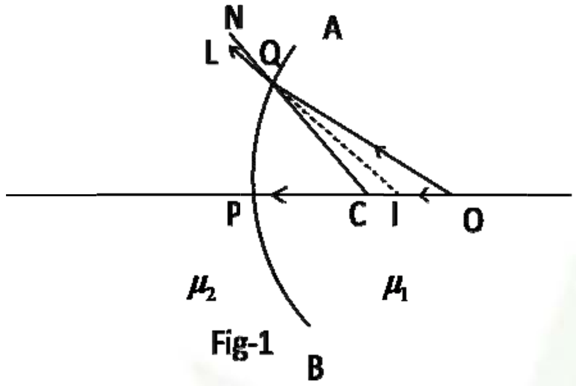

Fig-1

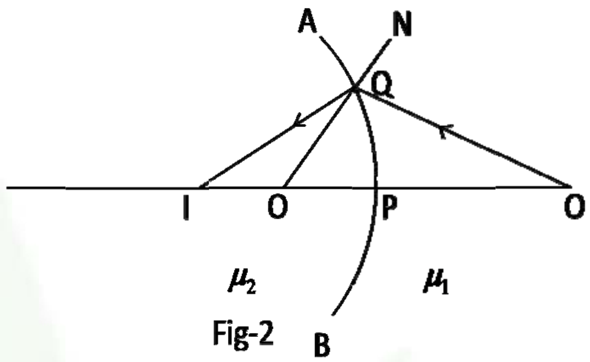

Fig-2

AB is a spherically curved surface, separating the two media of refractive indices & μ_2 $(\mu_2 > \mu_1)$. In figure-1 the concave surface faces the rarer medium & in figure-2 μ_1 the convex surface faces the rarer medium. A point object O is taken in the rarer medium on the principal axis.

Ray diagram: A ray from O is incident at any point Q and after refraction bends towards the normal CQN ($\mu_2 > \mu_1$) and goes along QL. Another ray from O is incident along the principal axis OCP & since it is incident normally it passes out without any deviation. These two refracted rays meet at I directly in figure-2 and by producing back in figure-1. Hence I is the image of the object O formed by refraction at the curved surface.

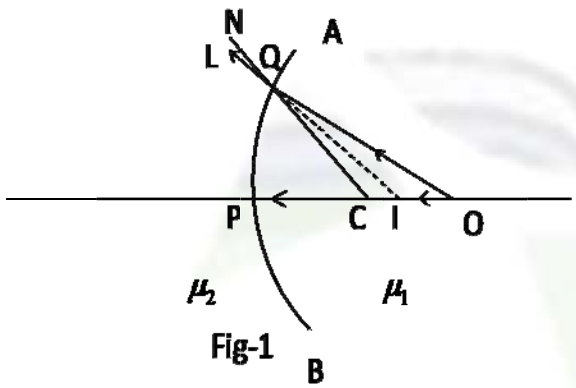

Fig-1

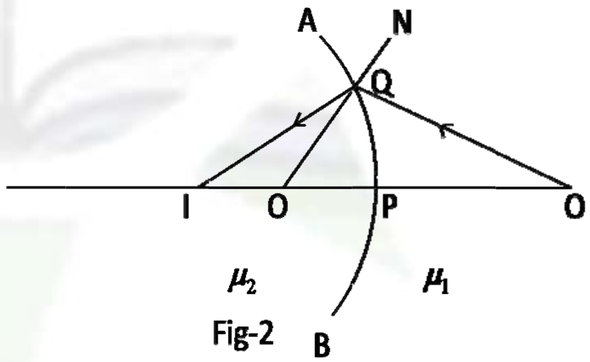

Fig-2

Calculation:

$$Let\ \angle OQC = \alpha, \angle IQC = \beta\ \&\ \angle OCQ = \gamma$$

Applying Snell's law

In figure - 1

$$\mu_1 \sin\alpha = \mu_2 \sin\beta$$

$$\mu_1 \frac{\sin\alpha}{\sin\gamma} = \mu_2 \frac{\sin\beta}{\sin\gamma} \to (1)$$

In figure - 2

$$\mu_1 \sin(180 - \alpha) = \mu_2 \sin\beta$$

$$\mu_1 \frac{\sin\alpha}{\sin\gamma} = \mu_2 \frac{\sin\beta}{\sin\gamma} \to (2)$$

Thus for both the figure the relation is same.

$$\text{Fig - 1}, \Delta OCQ: \frac{OC}{\sin\alpha} = \frac{OQ}{\sin\gamma} \text{ or } \frac{OC}{OQ} = \frac{\sin\alpha}{\sin\gamma}$$

$$\text{From } \Delta ICQ: \frac{IC}{\sin\beta} = \frac{IQ}{\sin\gamma} \text{ or } \frac{IC}{IQ} = \frac{\sin\beta}{\sin\gamma}$$

Putting the values in equation(2)

$$\mu_1 \frac{OC}{OQ} = \mu_2 \frac{IC}{IQ} \rightarrow (3)$$

For figure - 2 : From ΔOCQ using properties of tringle

$$\frac{OC}{\sin\alpha} = \frac{OQ}{\sin\gamma} \text{ or } \frac{OC}{OQ} = \frac{\sin\alpha}{\sin\gamma}$$

From ΔOCQ :

$$\frac{IC}{\sin\beta} = \frac{IQ}{\sin(180-\gamma)} \text{ or } \frac{IC}{IQ} = \frac{\sin\beta}{\sin\gamma}$$

Putting these values in equation (3)

$$\mu_1 \frac{OC}{OQ} = \mu_2 \frac{IC}{IQ} \rightarrow (4)$$

From equation (3) and (4) we see that for both figures relation is same.

Since the points P & Q are close

$\therefore OQ \approx OP$ & $IQ \approx IP$

For both the figures the relation is

$$\mu_1 \frac{OC}{OP} = \mu_2 \frac{IC}{IP} \rightarrow (5)$$

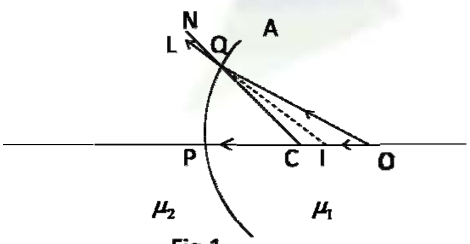

Fig-1

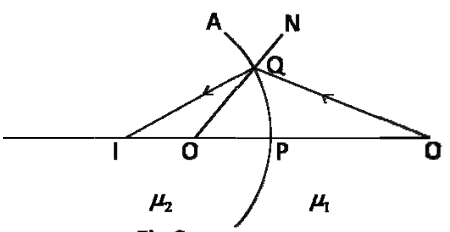

Fig-2

For figure - 1 for concave surface

PO = Object distance = u(say) = Positive

PI = Image distance = v(say) = Negative

PC = Radius of curvature = r(say) = Negative

OC = OP - PC = u - (-r) = u + r

IC = IP - PC = -v - (-r) = r - v

Putting the values in equation (5)

$$\mu_1 \left[\frac{u+r}{u}\right] = \mu_2 \left[\frac{r-v}{v}\right]$$

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r} \rightarrow (6)$$

For figure(2) for convex surface :

PO = object distance = u (say) = Positive

PI = Image distance = v(say) = Positive

PC = Radius of curvature = r(say) = Positive

OC = OP + PC = u + r, IC = IP - PC = v - r

From equation (5) :

$$\mu_1 \left[\frac{u+r}{u}\right] = \mu_2 \left[\frac{v-r}{v}\right]$$

$$\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{r} \rightarrow (7)$$

From equation (6) and (7) we find that for refraction at both concave and convex surface with object in the rarer medium the relation between the object distance & image distance is given by

$$\frac{\mu_o}{u} + \frac{\mu_i}{v} = \frac{\mu_i - \mu_o}{r} \rightarrow (8)$$

μ_o & μ_i are refractive index of object medium ($\mu_o = \mu_1$) and image medium ($\mu_i = \mu_2$) respectively.

Object Medium: The medium in which the incident rays strike the surface is known as object medium.

Image Medium: The medium in which the refracted rays leave the surface is known as image medium.