Stokes Formula

Stoke studied the motion of a solid sphere (of small size) through a viscous fluid and made the following observations. The viscous force of the medium through which the sphere falls depends on

(i) Size of the sphere

$F \propto r^a$ Where a is unknown dimensional coefficient

(ii) Speed of the body through the medium, greater is the speed higher is the viscous force

$F \propto v^b$ Where b is unknown dimensional coefficient

(iii) Nature of liquid i.e. coefficient of viscosity of the liquid

$F \propto \eta^c$ Where b is unknown dimensional coefficient

$$\therefore F \propto r^a v^b \eta^c$$
$$F = K r^a v^b \eta^c$$

Where K is dimensionless constant of proportionality.

The unknown dimensional coefficient can be calculated by dimensional equation method

$$[MLT^{-2}] = [L^a][LT^{-1}]^b [ML^{-1}T^{-1}]^c$$
$$[MLT^{-2}] = [M^c][L^{a+b-c}][T^{-b-c}]$$

comparing the dimensional coefficient of M, L & T

from both sides of equation respectively

$1 = c$

$2 = (b + c) \because c = 1$

$\therefore 2 = b + 1$

$b = 1$

$1 = a + b - c$

putting $c = 1, b = 1$

$\therefore a = 1$

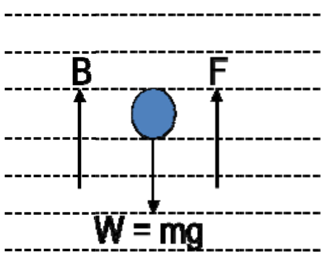
Putting the values in equation (1) F = kηrv

Stoke from his experimental data calculated the value of k and found it to be 6π

$$F = 6\pi\eta r v$$

Equation is known as Stokes formula.

Theory: Let us consider a small sphere of diameter about 5 mm falling freely through a viscous medium which is the experimental liquid



Given

r = radius of the sphere

ρ = density of the material of the sphere

σ = density of the experimental liquid

η = coefficient of viscosity of the experimental liquid

Let v be the velocity of the sphere

The force acting on the sphere

(1) The weight w=mg acting along vertically downward direction

(2) The buoyant force B acting along vertically upward direction

(3) The viscous force F acting along opposite to the direction of motion i.e. along vertically upward direction.

Using stokes formula:

$$F = 6\pi\eta r v = K.v \longrightarrow (1)$$

$$\text{where } K = 6\pi\eta r \longrightarrow (2)$$

Hence the resultant downward force on the sphere

R = W – B – F

$$R = W - B - K v \longrightarrow (3)$$

Initially v = 0, R = W – B > 0 , thus there is a resultant downward force due to which the sphere starts falling with an acceleration and the velocity v increases continuously. As v increases R decreases continuously and becomes zero. The moment R vanishes Acceleration becomes constant, the sphere continues to fall with constant velocity (v_t) known as terminal velocity.

$$0 = W - B - Kv_t$$
$$Kv_t = W - B \longrightarrow (4)$$

Thus when V = v_t , then R = 0

W = volume x density x g $= \frac{4}{3}\pi r^3 \rho g \longrightarrow (5)$

B = Weight of equal volume of liquid $= \frac{4}{3}\pi r^3 \sigma g \longrightarrow (6)$

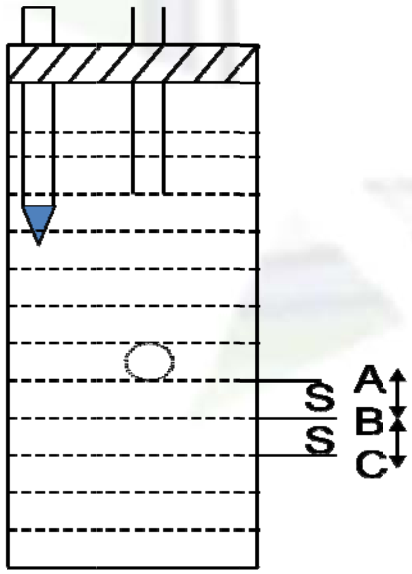
Putting equation (2), (5) & (6) in (4) we get

$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$
$$\eta = \frac{2r^2 g(\rho - \sigma)}{9v_t} \longrightarrow (7)$$

Using equation (7) coefficient of viscosity can be calculated.

Experimental arrangement: Few steel balls of suitable size whose maximum diameter depends on the nature of the experimental liquid are selected.

The diameters of these steel balls are measured by a screw gauge.



The experimental liquid is taken in tall jar whose upper end is closed with a cock. Three marks A,B & C are given on the glass jar at equal spacing. AB = BC = S

The first mark A is given well below the free surface of the liquid so that the steel balls attain terminal velocity before reaching the mark A. The time taken by the steel ball to cover AB & BC are measured separately by stop clocks, if the recorded time are nearly same within the experimental error we conclude that velocity is constant.

$$v_t = \frac{s+s}{t_1 + t_2} \quad \left\{ \quad (t_1 \approx t_2) \right.$$

Thus knowing r, v_t, ρ and σ, η can be calculated from equation (1) and this is the coefficient viscosity at the recorded temperature.

Correction: Ladenburg's correction

In deriving equation (7) two assumptions are made

(i) The liquid is continuous

(i i) The liquid is in infinite extent

However in the experiment the liquid is bounded by walls and hence a correction term must be applied. The correction expression is $$\eta = \frac{2}{9}\frac{r^2 g(\rho - \sigma)}{v_t\left[1 + 2.4\frac{r}{R}\right]\left[1 + 3.3\frac{r}{h}\right]}$$

Where R is the radius of the cylindrical jar h = height of the liquid column in the jar.