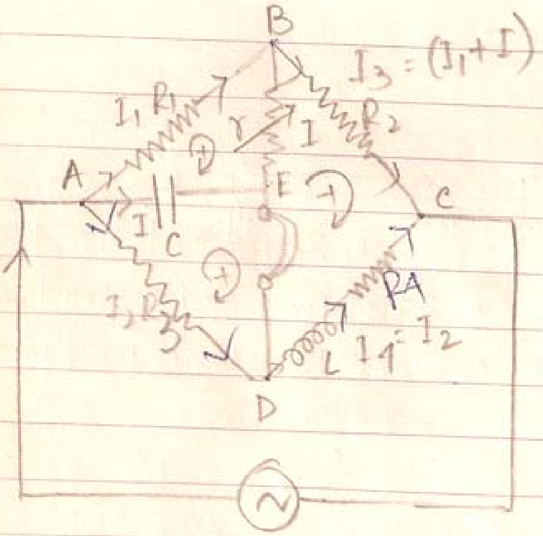
Anderson's bridge: This bridge is used to measure the self-inductance of a coil 'L'. There are other bridges also such as Owen's bridge; Maxwell's bridge etc; for determination of 'L' but Anderson's bridge is most accurate.



The circuit connection is made as shown in the diagram. The distribution of current in different branches are as shown.

Applying Kirchoff's second law to the closed mesh ABE

$$I_1 R_1 - I r - I X_C = 0$$

$$\text{or } I_1 R_1 - I r - \frac{I}{j\omega C} = 0$$

$$\text{or } I_1 R_1 = I\left(r + \frac{1}{j\omega C}\right)$$

$$\text{or } I_1 = \frac{I}{R_1}\left[r + \frac{1}{j\omega C}\right] \quad \text{—— (1)}$$

Applying Kirchoff's second law, in the closed mesh AED

$$\frac{I}{j\omega C} - I_2 R_3 = 0 \quad \text{or } I_2 = \frac{I}{R_3 j\omega C} \quad \text{—— (2)}$$

Applying Kirchhoff's Second law in the closed mesh BCDB (7)

$$(I_1 + I) R_2 - I_2 (j\omega L + R_4) + I r = 0$$

putting the values of I_1 & I_2 from eq^n (1) & (2)

$$\frac{I}{R_1}\left(r + \frac{1}{j\omega C}\right) R_2 + I R_2 - \frac{I}{j\omega C R_3} j\omega L - \frac{I R_4}{j\omega C R_3} + I r = 0$$

$$\frac{I r R_2}{R_1} + \frac{I R_2}{R_1 j\omega C} + I R_2 - \frac{I L}{C R_3} - \frac{I R_4}{R_3 j\omega C} + I r = 0$$

Equating real and imaginary parts from both sides:

$$I r \frac{R_2}{R_1} + I R_2 - \frac{I L}{C R_3} + I r = 0$$

$$\text{or } \frac{L}{C R_3} = \left[R_2 + r\left(\frac{R_2}{R_1} + 1\right)\right]$$

$$\text{or } L = C R_2 R_3 + C R_3 r \left(\frac{R_2}{R_1} + 1\right) \quad \text{--- (3)}$$

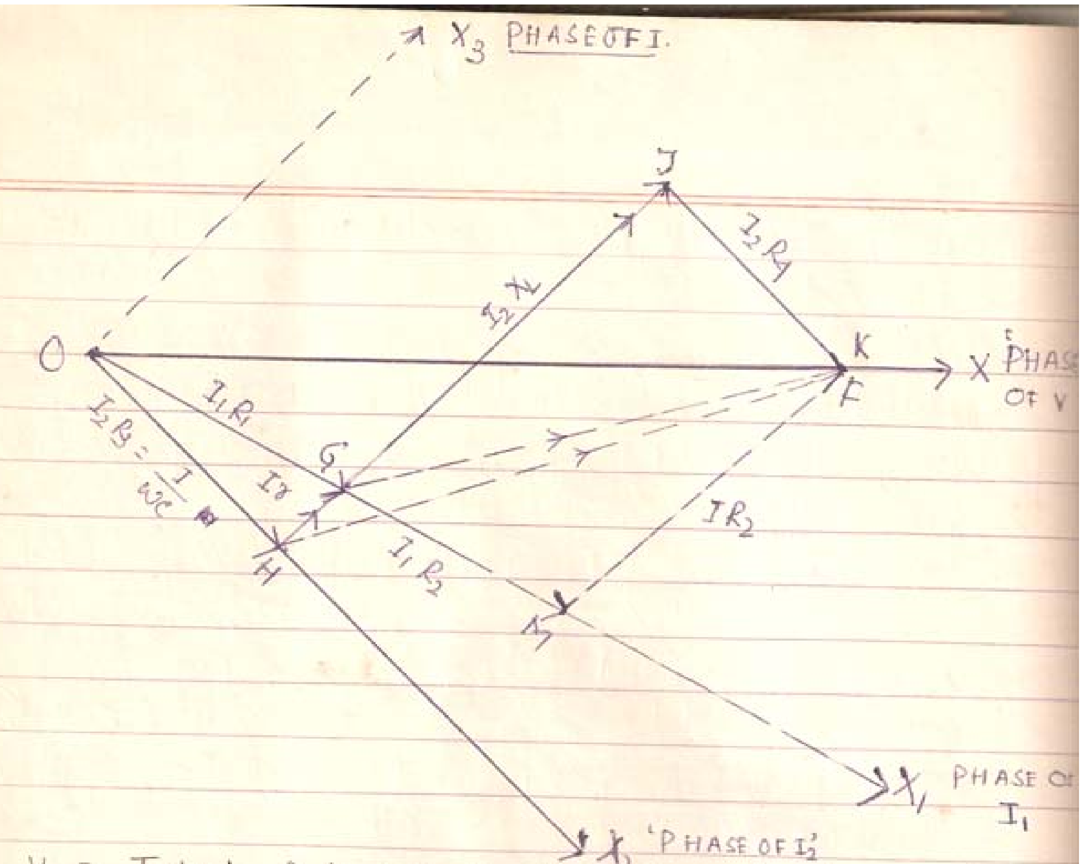
from eq^n (3) it is evident that the bridge can be balanced only if $L > C R_2 R_3$.

Equating imaginary part:

$$\frac{R_2}{R_1 \omega C} - \frac{R_4}{R_3 \omega C} = 0 \qquad \frac{R_2}{R_1} = \frac{R_4}{R_3} \quad \text{--- (4)}$$

We now draw the vector diagram of Anderson's bridge:-

(i) Let OX represent the phase of the total P.D across the circuit.



V = Total P.D across the circuit i.e the P.D between the points A & C (via B or D)

∴ V can be represented by a vector $\overrightarrow{OF}$ of length equal to V and direction along $\overrightarrow{OX}$.

Let $\overrightarrow{OX_1}$, $\overrightarrow{OX_2}$ & $\overrightarrow{OX_3}$ represent the phase of the current I_1, I_2 & I respectively.

(ii) The P.D across AB; $V_{AB} = I_1 R_1$ and this P.d is in phase with the current I_1 & hence can be represented by a vector $\overrightarrow{OG}$ of length $I_1 R_1$ and direction along $\overrightarrow{OX_1}$.

(iii) The P.D across AD; $V_{AD} = I_2 R_3$ being in phase with current I_2 can be represented by a vector $\overrightarrow{OH}$ of length $I_2 R_3$ and direction along OX_2.

(iv) P.D across DC = $V_{DC} = V_L + V_{R_4}$

$= I_2 j . X_L + I_2 R_4$

$V_L = I_2 j X_L$ Can be represented by a vector $\overrightarrow{HJ}$ of length $I_2 X_L$ and direction 90° ahead of $\overrightarrow{OX_2}$
$V_4 = I_2 R_4$ Can be represented by a vector $\overrightarrow{JK}$ of length $I_2 R_4$ and direction $||^{l}$ to OX_2.

$$\therefore \overrightarrow{HJ} + \overrightarrow{JK} = \overrightarrow{HK} = \overrightarrow{V_{DC}}$$

$$\therefore \overrightarrow{V_{AD}} + \overrightarrow{V_{DC}} = \overrightarrow{OH} + \overrightarrow{HK} = \overrightarrow{OK}$$

Since K and F are same points in the diagram

$$\overrightarrow{V_{AD}} + \overrightarrow{V_{DC}} = \overrightarrow{OH} + \overrightarrow{HK} = \overrightarrow{OK} = \overrightarrow{OF} = \overrightarrow{V}.$$

(v) P.D across the Capacitor $V_C = \frac{I}{j\omega C}$ & lags the current I by phase angle 90°. Also P.D across the Capacitor $V_{AE} = V_{AD}$ at Equilibrium

$$V_{AE} = \frac{I}{j\omega C} = I_2 R_3$$

Hence we find that I_2 must be 90° behind I. Hence in the vector diagram $I = \overrightarrow{OX_3}$ is drawn $\perp^r$ to $I_2 = \overrightarrow{OX_2}$.

$\therefore V_{AE} = I X_C = \overrightarrow{OH}$ i.e length of OH is $I/\omega C$ $= I_2 R_3$ & direction along $\overrightarrow{OX_2}$ $\perp^r$ to $\overrightarrow{OX_3}$.

(vi) P.D across r ; $V_{EB} = Ir$ and in phase with current I & Can be represented by a vector $\overrightarrow{HG}$ of length Ir & direction along $||^{l}$ to $\overrightarrow{OX_3}$ i.e $\perp^r$ to $\overrightarrow{OX_2}$ So that $\overrightarrow{V_{AE}} + \overrightarrow{V_{EB}} = \overrightarrow{OH} + \overrightarrow{HG} = \overrightarrow{OG} = \overrightarrow{V_{AB}}$.

(VII) P.D across BC $\vec{V}_{BC} = \vec{I}_3 R_2 = (\vec{I}_1 + \vec{I}) R_2$

or $\vec{V}_{BC} = \vec{I}_1 R_2 + \vec{I} R_2$

$\vec{I}_1 R_2 = \vec{GM}$ length of GM $= I_1 R_2$ and direction of $\vec{GM}$ is along $\vec{OX_1}$.

$\vec{I R_2} = \vec{MF}$, length of MF $= I R_2$ and direction of $\vec{MF}$ is along parallel to $\vec{OX_3}$.

$\vec{V}_{BC} = \vec{GM} + \vec{MF} = \vec{GF}$

$\therefore \vec{V}_{AB} + \vec{V}_{BC} = \vec{OG} + \vec{GF} = \vec{OF} = \vec{V}$ = total P.d across the circuit.

Q. Describe with theory Anderson's bridge method of determining the self inductance of a coil.