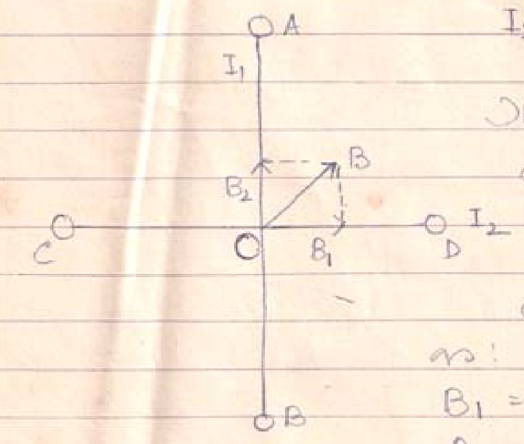
Qr. What is a rotating magnetic field? Describe the Construction of an induction motor. Calculate the torque produced in it?

Rotating magnetic field and Induction motor:

A rotating magnetic field is one in which flux rotates round a fixed axis. It can be produced by merely rotating a magnet or a current carrying coil. It can also be produced employing alternating current as follows.

Let us consider two coils carrying alternating current, they are placed at right angles to each other. Each coil will produce its own magnetic field. The coils at any point will have the same frequency as the currents.

$$I_1 = I_{o_1} \sin(\omega t + \theta)$$
$$I_2 = I_{o_2} \sin \omega t.$$



The magnetic fields due to each coil will be in phase with respect to the respective currents and may be represented as:

$$B_1 = B_{o_1} \sin(\omega t + \theta)$$
$$B_2 = B_{o_2} \sin \omega t.$$

If the field B_1 is due to the coil AB & B_2 the field due to the coil 'CD', they may be represented by Vector B_1 and B_2 respectively (as shown)

Let B' represent the resultant field at any instant then:

$$B' = \sqrt{B_1^2 + B_2^2}$$

& $\tan\phi = \frac{B_2}{B_1}$ where 'ϕ' angle between $\vec{B'}$ & $\vec{B_1}$.

The resultant field varies periodically as shown below.

At $t=0$, $B_2 = 0$ & $B' = B_1 = B_{01}\sin(+\theta)$

at $t = \frac{-\theta}{\omega}$; $B_1 = 0$; $B' = B_2 = B_{02}\sin(-\theta)$

at $t = \frac{\pi}{\omega}$, $B_2 = 0$; $B' = B_1 = B_{01}\sin\theta$.

at $t = \frac{\pi-\theta}{\omega}$; $B_1 = 0$; $B' = B_2 = B_{02}\sin(\pi-\theta) = B_{02}\sin\theta$.

This shows that the magnetic field rotates as well as its magnitude varies periodically.

If $B_{01} = B_{02} = B_0$

$$\therefore B' = B_0\left[\sin^2\omega t + \sin^2(\omega t+\theta)\right]^{1/2}$$

$$\text{and } \tan\phi = \frac{\sin\omega t}{\sin(\omega t+\theta)}$$

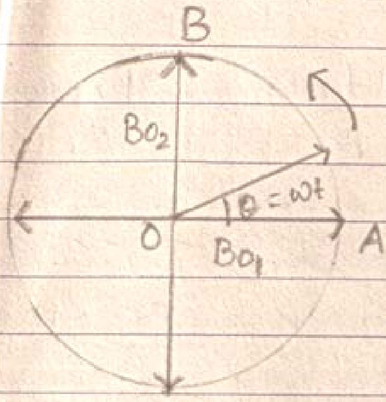
If in addition to the above B_1 leads B_2 by 90°

i.e. $\theta = \pi/2$, $B' = B_0\left[\sin^2\omega t + \cos^2\omega t\right]^{1/2} = B_0$

At $t=0$; $B_2 = B_{02}.\sin 0 = 0$; $B_1 = B_0$.

at $t = \frac{\pi}{2\omega}$; $B_1 = 0$, $B_2 = B_0$ & $B' = B_0$

Thus during the period from $t=0$ to $t = \frac{\pi}{2\omega}$ B' has rotated from the position OA to OB and direction of rotation of B' is thus anti clock wise when B_1 leads B_2 by 90° i.e. $\theta = \frac{\pi}{2}$



$\theta = -\pi/2$ then $B_1 = -B_0 \cos\omega t$

$B_2 = B_0 \sin\omega t$

at $t = 0$, $B_2 = 0$; $B_1 = -B_0$ & $B' = B_0$

at $t = \frac{\pi}{2\omega}$; $B_1 = 0$; $B_2 = B_0$ & $B' = B_0$

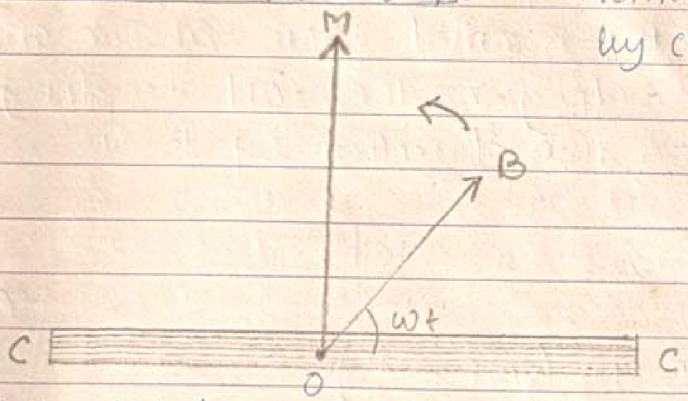
$$\tan\phi = -\frac{\sin\omega t}{\cos\omega t} = -\tan\omega t = \tan(-\omega t)$$

$$\boxed{\therefore \phi = -\omega t}$$

Thus the magnetic field rotates in negative direction i.e opposite to the direction of the vector B_{0_1} and B_{0_2}. Its angular velocity is $(-\omega)$.

Let us consider a conductor placed at O in a rotating magnetic field; induced current will be produced in the conductor which will stop the relative motion between the conductor and the field and the conductor also starts rotating in the direction of the resultant field. Thus a couple acts on the conductor placed in rotating magnetic field.

To calculate the couple :- Plane of the coil is represent by CC.



The total magnetic flux passing through the coil at any instant 't' is :

$$\phi_m = BA \sin \omega t$$

where A is the effective area of the coil and 'ωt' is the angle between B and the plane of the coil at any instant t.

The e.m.f induced in the coil when ϕ_m change

$$e = -\frac{d\phi_m}{dt} = -BA\omega \cos \omega t$$

$$= BA\omega \sin(\omega t - \pi/2)$$

The induced current in the coil :

$$I = \frac{BA\omega}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \frac{\pi}{2} - \alpha)$$

where L = Inductance of the coil.

R = Resistance of the coil

& α = is given by $\tan \alpha = \frac{\omega L}{R}$

The magnetic moment of coil is M = AI and perpendicular to the plane of the coil being directed along the normal OM. The moment of the couple acting on the coil tending to turn it into the direction of B is

$$\tau = M \times B$$

$$\text{or } \tau = MB \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -BAI \cos\omega t$$

$$\text{or } \tau = -\frac{B^2 A^2 \omega}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(\omega t - \frac{\pi}{2} - \alpha\right) \cos\omega t$$

$$= \frac{B^2 A^2 \omega}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \alpha) . \cos\omega t$$

$$= \frac{B^2 A^2 \omega}{\sqrt{R^2 + \omega^2 L^2}} \left[\cos^2\omega t \cdot \cos\alpha + \sin\omega t \cdot \cos\omega t \cdot \sin\alpha\right]$$

Since the average of $\cos^2\omega t$ over one cycle is $\frac{1}{2}$ and that of $\sin\omega t . \cos\omega t$ or $\frac{1}{2}\sin 2\omega t$ is zero. The mean couple over one cycle acting on the coil is :

$$\tau = \frac{1}{2} \frac{B^2 A^2 \omega}{\sqrt{R^2 + \omega^2 L^2}} . \cos\alpha.$$

$$= \frac{1}{2} \frac{B^2 A^2 \omega}{\sqrt{R^2 + \omega^2 L^2}} . \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{1}{2} \frac{B^2 A^2 \omega R}{(R^2 + \omega^2 L^2)}$$

The direction of the mean couple would be the Same as that of the rotation of the field. The above eqn shows that the mean couple depends on the value of ω - the relative angular velocity of the field with respect to the coil. The average couple will be zero when ω = ∞ and also when ω = 0.

If then the coil is capable of rotating its angular velocity in the direction of the field will increase until the rate at which work is done against friction of all kind is equal to the rate of work done by the rotating field. When the coil is released its ang. vel. will increase at first. This will result in a decrease in 'ω' the relative ang. velocity of the field, with respect to the coil and the couple τ will further increase. The increase in the couple will result in a further increase in the angular velocity of the coil which will result in a further decrease in 'ω'. The ang. speed of the coil will however never be equal to that of the field as in that case ω = 0 and the couple τ will vanish. The average couple has maximum value for some value of 'ω' between 0 to ∞. This value of 'ω' can be found by diff. τ w.r.to ω & putting the result to zero.

$$\frac{d\tau}{d\omega} = \frac{B^2A^2R}{2}\frac{d}{d\omega}\left\{\frac{\omega}{(R^2+\omega^2L^2)}\right\}$$

$$= \frac{B^2A^2R}{2}\cdot\left[\frac{R^2+\omega^2L^2\cdot 1 - 2\omega^2L^2}{(R^2+\omega^2L^2)^2}\right]$$

$$\frac{d\tau}{d\omega} = \frac{B^2A^2R}{2} \cdot \frac{R^2 - \omega^2L^2}{(R^2+\omega^2L^2)^2}$$

putting this value equal to zero we get

$$\omega^2 L^2 = R^2 \quad \text{or} \quad \omega = \frac{R}{L}$$

diff. again w.r.t ω, we have:

$$\frac{d^2\tau}{d\omega^2} = \frac{B^2A^2R}{2} \frac{d}{d\omega}\left[\frac{R^2-\omega^2L^2}{(R^2+\omega^2L^2)^2}\right]$$

$$\text{or } \frac{d^2\tau}{d\omega^2} = \frac{B^2A^2R}{2}\left[\frac{(R^2+\omega^2L^2)^2(-2\omega L^2) - (R^2-\omega^2L^2)\ldots}{(R^2+\omega^2L^2)^4}\right]$$

$$\text{or } \frac{d^2\tau}{d\omega^2} = \frac{B^2A^2R}{2} \cdot \frac{2\omega L^2 \cancel{(R^2+\omega^2L^2)}\left[-(R^2+\omega^2L^2) - 2\ldots\right.}{(R^2+\omega^2L^2)^{\cancel{4}\,3}}$$

$$= \frac{B^2A^2R}{2}\left[\frac{2\omega^3L^4 - 6\omega L^2R^2}{(R^2+\omega^2L^2)^3}\right]$$

Putting $\omega = \frac{R}{L}$,

$$\frac{d^2\tau}{d\omega^2} = \frac{\left[2L^{\cancel{4}}\frac{R^3}{\cancel{L^3}} - 6\frac{R}{\cancel{L}}\cdot L^{\cancel{2}}R^2\right]}{(R^2+R^2)^3}\frac{B^2A^2R}{2}$$

$$= \frac{B^2A^2R}{2}\left[\frac{2R^3L - 6R^3L}{8R^6}\right] = \frac{B^2A^2R}{2}\left[-\frac{L}{2R\ldots}\right.$$

This is (–)ve. Thus τ is maximum when $\boxed{\omega = \ldots}$

Again $\tan\alpha = \frac{\omega L}{R} = \frac{L}{R}\cdot\frac{R}{L} = 1$ $\boxed{\alpha = 45^\circ}$

Therefore the value of 'α' for max^m couple is 45° and the value of maximum couple

$$\tau_{max} = \frac{B^2A^2R \cdot R/L}{2\left(R^2 + \frac{R^2}{L^2}\cdot L^2\right)} = \frac{B^2A^2}{4L}$$

fig. ④ gives the plot of the quantity $\frac{\omega}{(R^2+\omega^2L^2)} = \frac{\omega}{Z^2}$ agai...