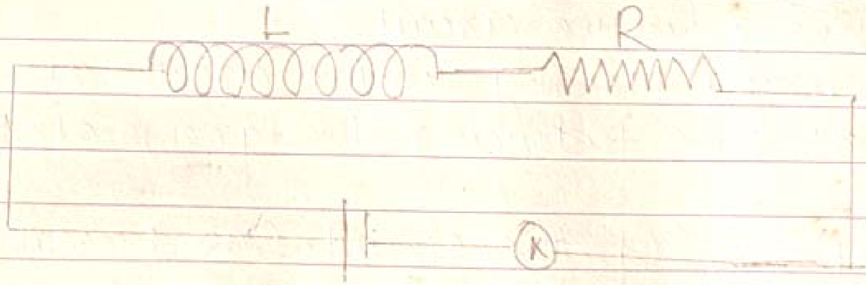
Transient (which exists for a very short interval



L – R Circuit :

Growth: An inductance and a resistance are connected in a series with a steady source of e.m.f through key K.
When the key is closed a constant unidirectional current flows through the coil. Just at the moment key is closed current in the circuit rises from 0 to that steady value & takes some finite time in this process. During this small time interval, the varying current flowing through the inductance coil induces an emf across the coil which according to Lenz's law opposes the cause i.e opposes the growth of current and hence the growth of current is delayed. The current takes a longer value time to reach to its final value. The varying current and the induced e.m.f exist for a very short time interval till the current attains the steady value and hence are known as transient.

Analysis :- Given : L = Coeff. of self induction of the coil.

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R = The total Ohmic resistance in the circuit.
E = The e.m.f of the Source.
I_0 = Steady or the final or the maximum current flowing in the circuit.

Let i = current flowing in the circuit at an instant of time t during the growth of current

The emf eq^n of the circuit can be written as

$$L\frac{di}{dt} + iR = E \quad \text{--- (1)} \quad \text{or } \frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$$

$$\text{or } \frac{di}{dt} = -\frac{R}{L}\left(i - \frac{E}{R}\right) \quad \text{--- (2)}$$

When $i = I_0$; $\frac{di}{dt} = 0$ from (1) ; $I_0R = E$

$$\text{or } \frac{E}{R} = I_0 \quad \text{--- (3).}$$

putting eq^n (3) in (2) and re-arranging :-

$$\frac{di}{(i - I_0)} = -\frac{R}{L}dt \quad \text{Integrating both side}$$

$$\log(i - I_0) = -\frac{R}{L}t + A \quad \text{--- (4).}$$

Where A is unknown const. of integration
Applying initial condition at $t = 0$; $i = 0$
from (4) :

$$\log(-I_0) = -\frac{R}{L}.0 + A \quad \therefore A = \log(-I_0) \quad \text{--- (5)}$$

putting eqⁿ ⑤ in ④ :-

$$\log(i - I_0) = -\frac{R}{L}t + \log(-I_0)$$

$$\log\left(\frac{i - I_0}{-I_0}\right) = -\frac{R}{L}t$$

$$\text{or } \log\left(1 - \frac{i}{I_0}\right) = -\frac{R}{L}.t$$

$$\left(1 - \frac{i}{I_0}\right) = \left(e^{-\frac{R}{L}t}\right)$$

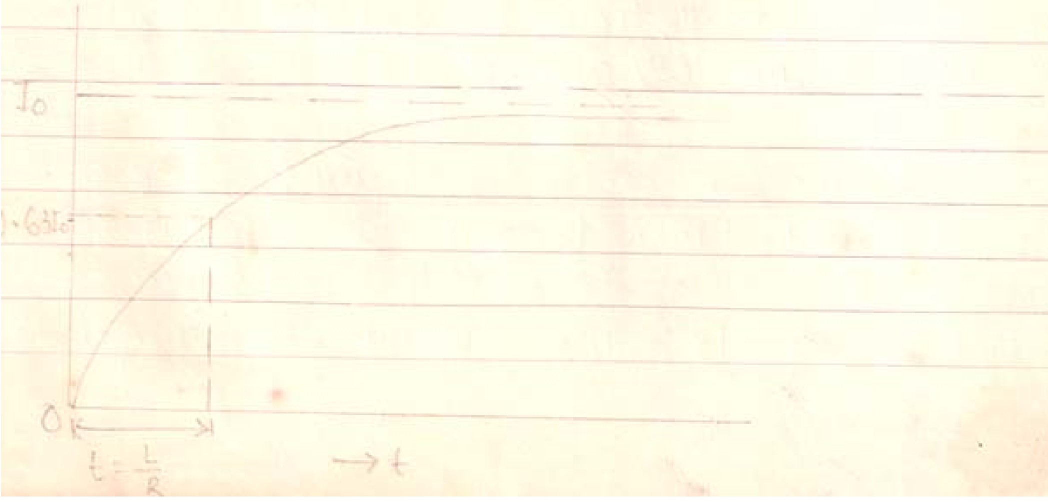
(By Taking anti-log): $\frac{i}{I_0} = 1 - e^{-\frac{R}{L}.t}$

$$\text{or } i = I_0\left[1 - e^{-\frac{R}{L}t}\right] \quad — ⑥$$

Since I_0, R & L are known the current in the circuit at any instant 't' during the growth can be calculated.

Discussions:

① From eqⁿ ⑥ we find that $i = I_0$ at $t = \infty$. Hence the current grows in the circuit in an exponential fashion and is asymtotic to the line $i = I_0$.



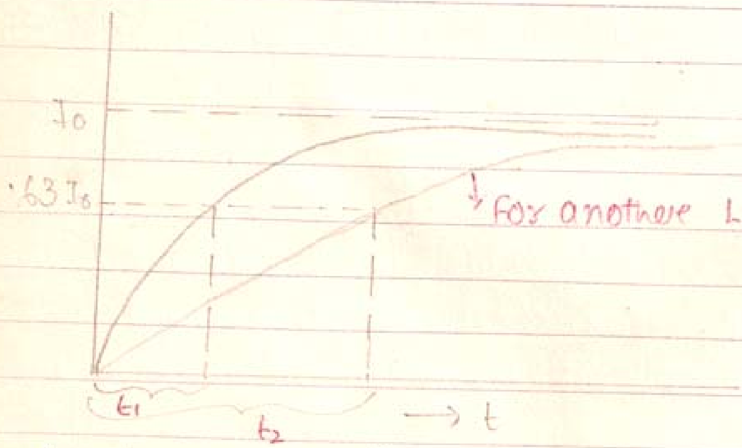
② At $t = \frac{L}{R}$ = const. from ⑥ $i = I_0\left[1-e^{-\frac{R}{L}\cdot\frac{L}{R}}\right]$

or $i = I_0\left[1-e^{-1}\right] = I_0\left[1-\frac{1}{2.7306}\right] = I_0\left[1-0.37\right]$

or $i = 0.63 I_0$ i.e if $I_0 = 100$ amp. $i = 63$ amp

Hence the constant $t = \frac{L}{R}$ is known as time constant of the L-R circuit and is defined as the time during which which current grows to 63% of its maximum value.

Physical Significance: Time constant of a circuit gives an idea about the rate of growth of current in the L-R circuit. Smaller is the time constant faster is the rate of growth of current because



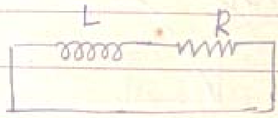
during that small time the current goes near to its final value. (63% of I_0).

Decay of current in L-R circuit:- Let a steady current flow through an L-R circuit when the key is taken off (i.e source of e.m.f is removed), the current in the circuit reduces from that Steady value to zero and takes some finite time in this process. During this fall of current the current is varying and this varying current flowing through the inductance produces an induced e.m.f, which opposes the cause i.e the decay of current. Hence the decay of current in the circuit is delayed.

I_0 = Initial current flowing in the circuit.

Let i = The current flowing in the circuit at any instant of time 't' during the decay of current.

The e.m.f eq^n of the circuit :



$$L\frac{di}{dt} + iR = 0$$

or $$\frac{di}{dt} = -\frac{R}{L}i$$

or $$\frac{di}{i} = -\frac{R}{L}dt$$ Integrating both Sides

$$\log i = -\frac{R}{L}t + B \quad \text{--- (7)}$$ where B is unknown Constant of Integration.

Applying the initial Condition at $t = 0$; $i = I_0$

$$\log I_0 = -0 + B \quad \text{or} \quad B = \log I_0 \quad \text{--- (8)}$$

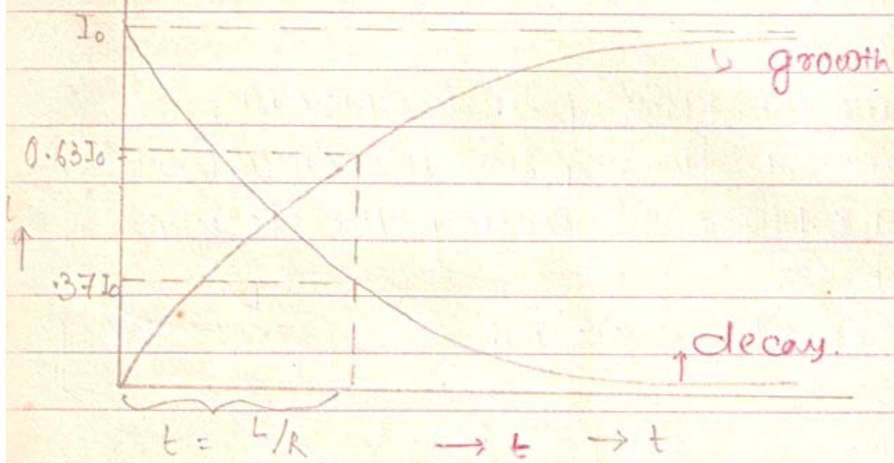
putting eq^n (8) in (7) : $$\log i = -\frac{R}{L}t + \log I_0$$

or $$\log\frac{i}{I_0} = -\frac{R}{L}t$$

or $$i = I_0 e^{-\frac{R}{L}t} \quad \text{--- (9)}$$

Using eq^n (9) we Can find the current flowing in the circuit at any instant 't' during the decay of current.

Discussions: ① from eqⁿ ⑨ we find that the current falls to zero value at $t = \infty$; i.e at $t = \infty$, $i = 0$. Thus the current in the circuit decreases in an exponential fashion and is asymptotic to $i = 0$.



② At $t = \frac{L}{R}$; from ⑨ $i = I_0 e^{-\frac{R}{L} \cdot \frac{L}{R}} = I_0 e^{-1}$

or $i = 0.37 I_0$

$t = L/R$ = const. = time constant.

Hence the time constant of an L-R circuit can also be defined as the time during which the current falls to 37% of its max^{m} value.