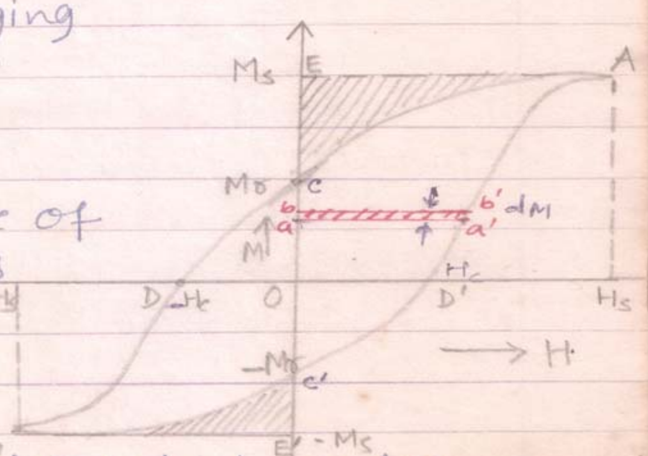
The energy Loss per unit volume of the Specimen per cycle of Magnetisation :

When a ferromagnetic Specimen is taken through cycles of magnetisation; the molecular magnets or atomic dipoles of the material get aligned & re-aligned in the direction of the magnetising field and the molecular motion involved in this process of alignment results in the production of heat due to friction and hence energy is lost.

We now calculate the loss in energy per unit volume of the specimen, per cycle of magnetization which is equal to the area of B-H curve or equal to μ_0 times the area of M-H Curve :-

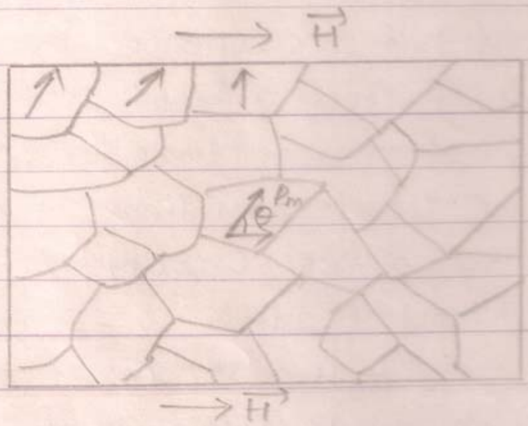
Let us consider a unit volume of a Specimen which is taken through one complete cycle of magnetisation by varying magnetising field from H_s to zero, zero to $-H_s$, $-H_s$ to zero and zero to $+H_s$ (by changing current through the Solenoid wound over the Specimen).



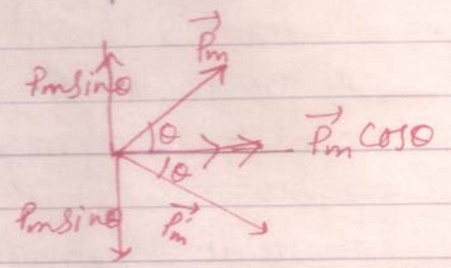
Let n = The number of ferromagnetic domains present in unit volume of the Specimen.

P_m = The magnetic moment of a domain selected at random.

Let us consider an instant, when the magnetising field is H and the magnetisation is M. Let at that instant the magnetic moment vector $\vec{P_m}$, make an angle 'θ' with the direction of the magnetising field.



Resolving the magnetic moment vectors parallel and perpendicular to the magnetising field, for all the magnetic domains present in the unit volume.



$$\sum p_m \cos\theta = M$$

$$\sum p_m \sin\theta = 0$$

The Summation is being taken over all the domains present in the Specimen.

or $\sum p_m \cos\theta = M$ —— (1)

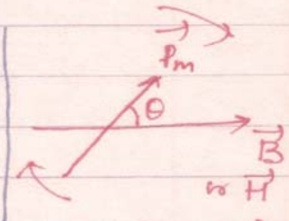
Differentiating eqn no (1) :-

$$dM = -\sum p_m \sin\theta \, d\theta \quad —— (2)$$

The torque acting on that magnetising domain of moment p_m, making an angle 'θ' with the magnetising field

$$C = p_m B \sin\theta = p_m \mu_0 H \sin\theta$$

In order to increase the magnetisation of the specimen; the magnetic moment vectors of the domains are to be brought parallel to the magnetising field i.e 'θ' is to be decreased.



$\because \vec{B} = \mu_0 \vec{H}$

In order to decrease θ, from θ to '$\theta - d\theta$' in presence of the couple

work done $= C(-d\theta)$

or $dW = -p_m \mu_0 H \sin\theta \, d\theta$

The negative sign indicates the work to done in favour of the couple.

Total workdone for all the domains, present in unit volume,

$$W = \sum dW = \sum -\mu_0 H p_m \sin\theta \, d\theta$$

putting eq^n (2) :-

$$\boxed{W = \mu_0 H dM} \quad \text{— (3)}$$

Eq^n (3) gives the work done to increase the magnetisation from 'M to 'M+dM'

From the graph; $\boxed{HdM = \text{area of the strip } abb'a'}$

Hence the work done to increase the magnetisation of the specimen from ~~Mr to Ms~~ M to 'M+dM'

$$W_1 = \int_{-M_r}^{+M_s} \cancel{\mu_0} HdM = \cancel{\mu_0 \times \text{area } C'AEC'}$$

$W = \mu_0 \times$ area of the strip $abb'a'$ —— (4)

Following eqⁿ (3) and (4) we can write the following expression.

work done to change the magnetisation of the specimen from $-M_r$ to M_s.

$$W_1 = \int_{-M_r}^{+M_s} \mu_0 H dM = \mu_0 \times \text{area } C'AEC' \text{ —— (5)}$$

The work done in changing the magnetisation from $+M_s$ to $+M_r$:-

$$W_2 = \int_{+M_s}^{+M_r} \mu_0 H dM = \mu_0 \times \text{area of the graph } AECA \text{ (6)}$$

From eqⁿ no: (5) and (6) we get that the energy lost (per unit volume of the specimen) when the specimen is taken through half cycle of magnetisation i.e 0 to H_s and H_s to 0 i.e M changes from $-M_r$ to $+M_r$

$$W' = \mu_0 \int_{-M_r}^{+M_s} H dM + \mu_0 \int_{+M_s}^{+M_r} H dM = \mu_0 \int_{-M_r}^{+M_r} H dM$$

$$= \mu_0 \times \text{area } C'ACC' \text{ —— (7)}$$

Similarly, the work done i.e energy lost when the specimen is taken through the other half cycle of magnetisation i.e 0 to $-H_s$ & $-H_s$ to 0 or M changes from $+M_r$ to $-M_s$ & $-M_s$ to $-M_r$

$$W'' = \mu_0 \int_{+M_r}^{-M_s} H dM + \mu_0 \int_{-M_s}^{-M_r} H dM = \mu_0 \int_{+M_r}^{-M_r} H dM = \mu_0 \times \text{area } CA\ldots \text{ —— (8)}$$