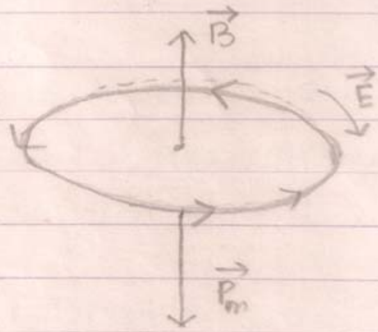
Langevin's Theory Of Diamagnetism:

The negative magnetic moment induced in matter by external magnetic field and the consequent negative susceptibility of diamagnetic substance can be explained on the basis of Langevin's theory.

Let us consider an electron rotates in a circular orbit in an anticlockwise direction as shown.



e = Charge of the electron
r = radius of the orbit.
V_0 = Initial velocity of the electron when no external field is applied.

$V_0 + \Delta V$ = The final orbital velocity of the electron when the external field is applied.
B = The induction vector of the externally applied magnetic field.

As the electron rotates in the orbit it gives rise to a current in a circular (orbit) loop having direction opposite to that of rotation of the electron and the magnetic moment $\vec{P_m}$ due to this circular current loop is found to be along downward direction in the fig. perpendicular to the plane of the loop. [As obtained by curling the figures of right hand in the direction of current the thumb gives the direction of the magnetic field or moment]

During the interval, the applied magnetic field increases from 0 to B the magnetic flux ($\phi = \vec{B}\cdot\vec{A}$) through the loop changes & gives rise to an induced e.m.f given by Faraday's law:

$$\mathcal{E} = -\frac{d\phi}{dt} = \frac{d}{dt}(BA) \quad \text{---} (1)$$

The direction of this induced e.m.f is given by Lenz's law.

According to Lenz's law the cause of induced e.m.f i.e the increase in B should be opposed. This is possible if $\vec{P_m}$ which is opposite to the direction of $\vec{B}$ increases. $\vec{P_m}$ will increase if current in the loop increases i.e the motion of the electron in the orbit becomes faster. Hence we require an electric field in a direction opposite to the direction of rotation of the electron so that electron may get accelarated.

Let $\vec{E}$ be the intensity of the electric field produced by the induced e.m.f

$$\therefore \quad \mathcal{E} = \oint \vec{E}\cdot\vec{dl} = E \cdot 2\pi r.$$

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$$\therefore \boxed{E = \frac{\mathcal{E}}{2\pi r}} \quad \text{—— (2)}$$

Force experienced by the electron in this field

$$m \cdot \frac{dv}{dt} = E \cdot e = \frac{e}{2\pi r} \cdot \mathcal{E} = \frac{e}{2\pi r} \frac{d\phi}{dt}$$

$$\text{or, } \frac{dv}{dt} = \frac{e}{m \cdot 2\pi r} \frac{d(AB)}{dt} = \frac{e \cdot \cancel{\pi} r^{\cancel{2}}}{m \cdot 2\cancel{\pi}\cancel{r}} \frac{dB}{dt}$$

$$\text{or, } \boxed{\frac{dv}{dt} = \frac{er}{2m} \frac{dB}{dt}} \quad \text{—— (3)}$$

The total change in velocity of the electron as the external field increases from 0 to B can be obtained by integrating (3):

$$\int_{v_0}^{v_0+\Delta v} dv = \frac{er}{2m} \int_0^B dB$$

$$\text{or, } \Delta V = \frac{er}{2m} \cdot B \quad \text{—— (4)}$$

Consequent change in frequency of rotation

$$\boxed{\Delta \omega_L = \frac{\Delta V}{r} = \frac{e}{2m} \cdot B} \quad \text{—— (5)}$$

This change in frequency is known as Larmor's frequency or Larmor's ang. velocity.

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We now show that ; the radius of the orbit of the electron remains unaltered by the effect of external magnetic field :
Since the centrepetal force required for uniform circular motion of the electron is supplied by Columb force of attraction of the nucleus

$$F_{columb} = \frac{1}{4\pi\epsilon_0}\frac{(Ze)e}{r^2} = \frac{mv_0^2}{r} = mr\omega_0^2 \quad (6)$$

Where ω_0 is the initial angular ~~frequency~~ velocity of the electron in the orbit.
When the external magnetic field of induction vector $\vec{B}$ is applied ; the angular frequency ω_0 changes to $\omega_0 \pm \Delta\omega_L$ [+ve or (-)ve depends on direction of the flow] Hence the linear velocity of the electron in the orbit changes to $v = r[\omega_0 \pm \Delta\omega_L]$
When the magnetic field $\vec{B}$ has been established the magnitude of centrepetal force needed is

$$F_c' = mr[\omega_0 \pm \Delta\omega_L]^2$$

$$F_c' = mr[\omega_0^2 \pm 2\omega_0\Delta\omega_L + \Delta\omega_L^2]$$

If $\vec{B}$ is not very very high, $\Delta\omega_L$ will be very small compared to ω_0 & hence $\Delta\omega_L^2$ can be neglected.

$$\boxed{F_c' = mr\omega_0^2 \pm 2mr\omega_0\Delta\omega_L} \quad (7)$$

Eqn (7) indicates that the columb force must change by an amount, $2mr\omega_0\Delta\omega_L$.

But we know that a

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Charge moving at const. speed in a uniform magnetic field in a direction at right angles to the field experience a force $[\vec{F} = -e(\vec{v} \times \vec{B})]$ whose magnitude is constant $(= evB)$ & direction is always perpendicular to the velocity & this force keeps the particle in uniform circular motion & supplied the reqd. centripetal force. Thus this force due to the magnetic field on the charged particle supplies the extra centripetal force $2mr\omega_0 \Delta\omega_L$, over and above the columb force.

Mag. force $= F_B = er[\omega_0 + \Delta\omega_L]B$

putting the value of B from eq^n(5):

$$F_B = \cancel{e}r[\omega_0 + \Delta\omega_L]\frac{2m}{\cancel{e}}\Delta\omega_L$$

Bev
evB
$er\omega B$
$er(\omega_0+\Delta\omega_L)B$

$$= 2mr\omega_0\Delta\omega_L + \underline{2mr\Delta\omega_L^2}$$

$$= 2mr\omega_0\Delta\omega_L$$ Same as extra centripetal force.

Now with the application of the external field $\vec{B}$ the velocity of the electron in the orbit changes and consequently the orbital magnetic moment also changes.

$$p_m = \frac{1}{2}evr = \frac{e}{2}r^2\omega$$

The change in orbital magnetic moment

$$\Delta p_m = \frac{1}{2}er^2\Delta\omega_L$$

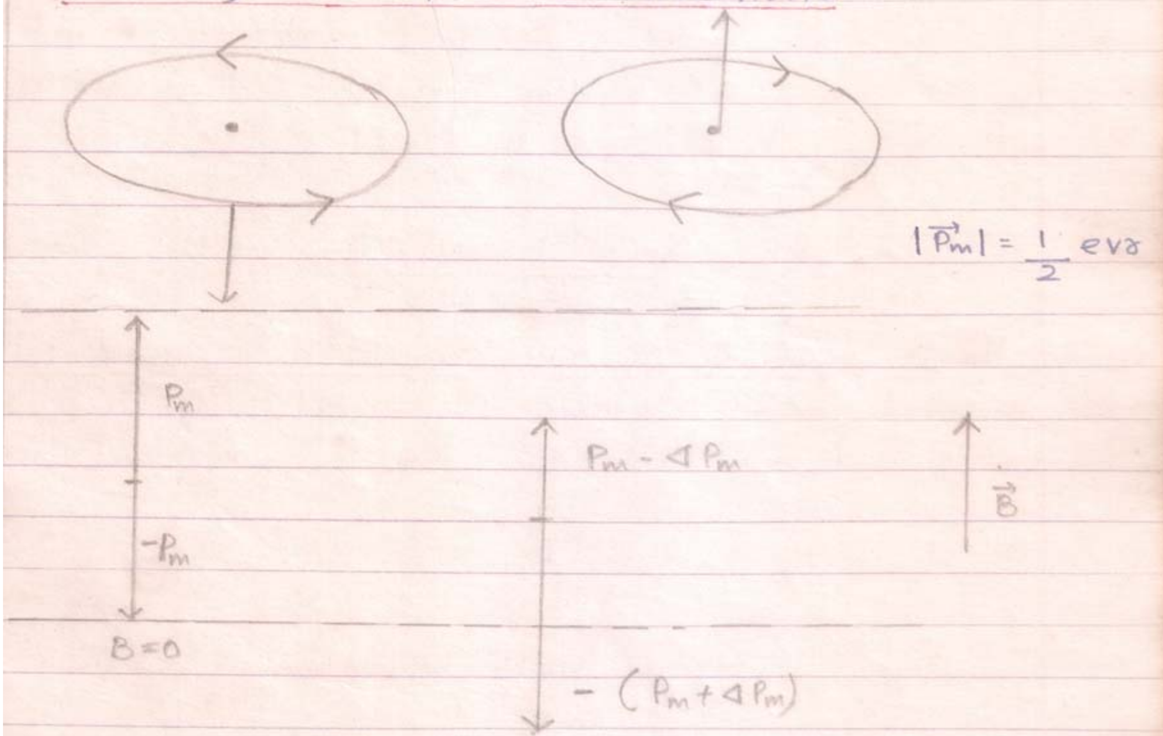
putting eq^n(5): $\Delta p_m = \frac{1}{2}er^2 . \frac{e}{2m}B = \frac{e^2r^2}{4m} . B$ — (8)

In vector form we can write eq^n (8) as:

$$\boxed{\Delta \vec{P}_m = -\frac{e^2 r^2}{4m}\vec{B}} \quad \text{——— (9)}$$

Where the (−)ve sign indicates that P_m decreases with increase in B in the direction of the field $\vec{B}$ & P_m increases with increase in B in a direction opposite to that of $\vec{B}$.

Let us now consider the case of two electrons rotating in opposite directions:



When no magnetic field is applied the orbital magnetic moment of the two electrons being equal and opposite will mutually cancel out. But in the presence of the external magnetic field $\vec{B}$ the magnetic moment in a direction of the field decreases while that in opposite direction increases

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giving rise to a resultant magnetic moment in a direction that opposite to the direction of the magnetic field.

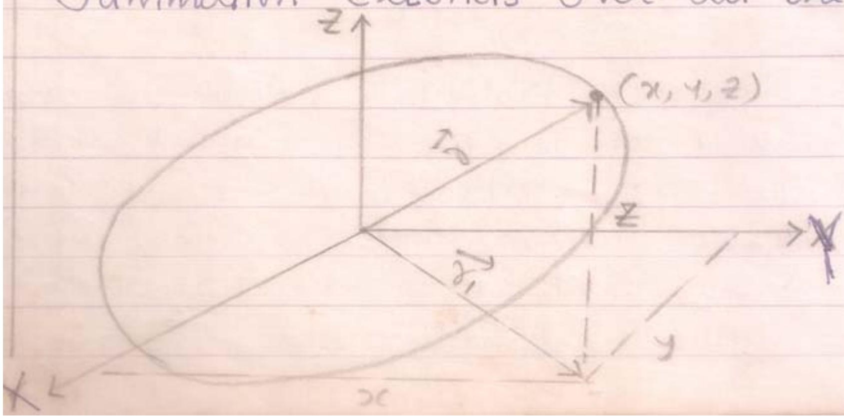
The eq^n (9) which has been derived for one electron can be extended to an atom having more than one electron. In deriving that eq^n we have assumed that the plane of the orbit is perpendicular to the field. However since the orbit can have any orientation w.r to the field the radius 'r' of the orbit in eq^n (9) should be replaced by r_1, where r_1 is the projection of r on the plane perpendicular to the field. Eq^n(9) then modifies to

$$\Delta \vec{P}_m = -\frac{e^2 r_1^2}{4m}\vec{B} \quad ----- (10)$$

There are a no. of randomly oriented orbits in the atom therefore the total magnetic moment induced in the atom by the external field

$$\vec{P}_m = -\frac{e^2 \vec{B}}{4m}\sum r_1^2$$

Summation extends over all the orbits.



Let the mean value of the radii of all possible orbits be $\vec{r}$ then

$$\overline{r}^2 = \overline{x}^2 + \overline{y}^2 + \overline{z}^2$$

For a spherically Symmetrical atom,

$$\overline{x}^2 = \overline{y}^2 = \overline{z}^2 = \frac{1}{3}\overline{r}^2$$

Moreover,

$$\overline{r_1}^2 = \overline{x}^2 + \overline{y}^2 = 2\overline{x}^2 = \frac{2}{3}\overline{r}^2$$

Therefore the total magnetic moment induced in the atom is

$$\overline{P_m} = -\frac{e^2\vec{B}}{4m}\sum r_1^2 = -\frac{e^2\vec{B}}{6m}\sum \overline{r}^2 \quad \text{---} (11)$$

The atomic susceptibility of the dia magnetic Substance

$$\chi_a = \frac{\text{Induced mag. moment per atom}}{\vec{H}}$$

$$= -\frac{e^2 B}{6mH}\sum \overline{r}^2$$

$$= -\frac{\mu_0 e^2}{6m}\sum \overline{r}^2 \quad \text{------} (12)$$

Let 'n' be the no. of atoms per unit volume of the substance, then the dia-magnetic susceptibility per unit volume

$$\chi_v = -\frac{n\mu_0 e^2}{6m}\sum \overline{r}^2 \quad \text{----} (13)$$