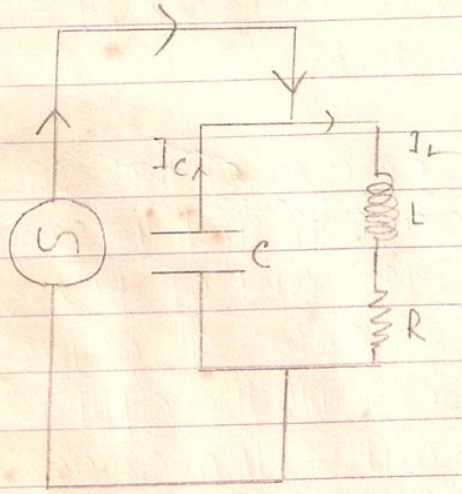
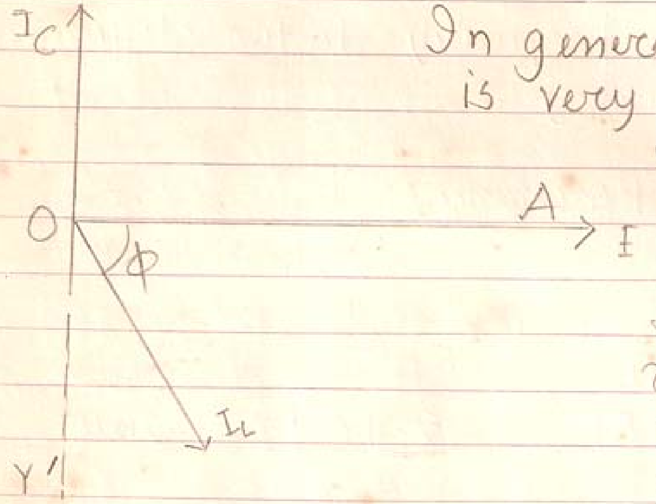
## Parallel Resonant Circuit :




Since the Source of e.m.f is connected $\parallel^{el}$ to both the components at any instant, the e.m.f across both the components will be same and current flowing through the two branches will be different. In the inductive branch the current lags the e.m.f

While in the Capacitive branch the current leads the e.m.f by phase angle $\pi/2$. But the current in the L-R branch lags the e.m.f by a phase angle '$\phi$' given by
$\tan\phi = \frac{\omega L}{R}$.




In general the resistance 'R' is very small & '$\tan\phi$' is very very high & $\phi \simeq \pi/2$

Thus in parallel resonant circuit, the current through the Capacitive branch & the inductive branch are almost 180° out of phase.

Resolving $I_L$ into two mutually perpendicular components along $\overrightarrow{OA}$ & $\perp^r$ to OA.

Component along OA, $I_L \cos\phi \simeq$ very small because $\phi \simeq \pi/2$

Component along OY' = $I_L \sin\phi$.

If the frequency of the applied e.m.f is such that the current $I_L \sin\phi$ is same as $I_C$, then they being 180° out of phase will mutually cancel out & only a very very small current in phase with the applied e.m.f flows in the circuit & the current is said to be in resonance with the applied e.m.f

Thus in parallel resonance; the current in the circuit is minimum in contrast with the series resonance, where the current is maximum, but in both the cases, the current in the circuit is in phase with the e.m.f.

The vector equation of the current in the circuit can be written as

$$\vec{I} = \vec{I_L} + \vec{I_C} \quad \text{--- (1)}$$

Let $Z^*$ be the complex vector impedance of the circuit

$$\therefore \quad \vec{I} = \frac{\vec{E}}{Z^*}, \quad \vec{I_L} = \frac{\vec{E}}{R+j\omega L}$$

$$\therefore \quad \vec{I_C} = \frac{\vec{E}}{1/j\omega C}$$

$\therefore$ putting these values in Eqn (1)

$$\frac{\vec{E}}{Z^*} = \frac{\vec{E}}{R+j\omega L} + \frac{\vec{E}}{1/j\omega C}$$

$$\text{Or} \quad \frac{1}{Z^*} = \frac{1}{R+j\omega L} + j\omega C$$

$$\text{Or} \quad \frac{1}{Z^*} = \frac{(R-j\omega L)}{(R+j\omega L)(R-j\omega L)} + j\omega C$$

$$\text{Or} \quad \frac{1}{Z^*} = \frac{R-j\omega L}{R^2+\omega^2 L^2} + j\omega C$$

$$\text{Or} \quad \frac{1}{Z^*} = \frac{R - j\omega\left[L - R^2 C - \omega^2 L^2 C\right]}{R^2 + \omega^2 L^2} \quad \text{--- (2)}$$

For the current in the circuit to be minimum

$Z^*$ should be maximum i.e the admittance $\frac{1}{Z^*}$ should be minimum.

From eq$^n$ (2) admittance becomes minimum when the quadrature part vanishes.

Thus for parallel resonance

$$L - R^2C - \omega_r^2 L^2 C = 0$$

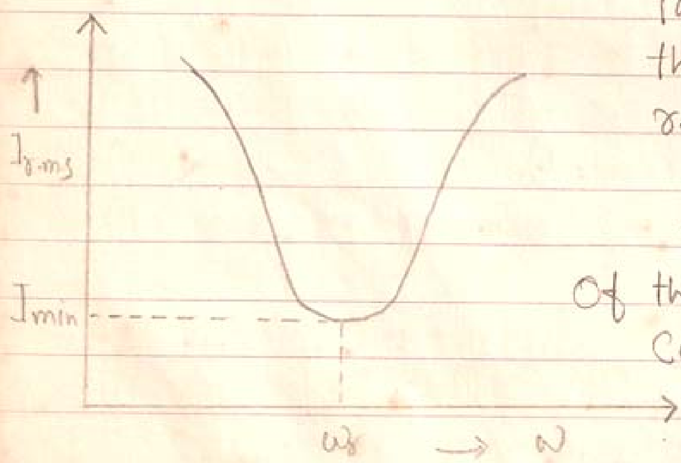
$$\text{or} \quad \omega_r^2 L^2 C = L - R^2 C$$

$$\text{or} \quad \omega_r^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\text{or} \quad \omega_r = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{--- (3)}$$

$$\text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{--- (4)}$$

Equation (4) gives the frequency of parallel resonance circuit.




From the graph we find that at $\omega = \omega_r$ the r.m.s current is minimum. From eq$^n$ (2) we find that the admittance of the circuit at resonance can be obtained by putting the quadrature component equal to zero.

i.e $\frac{1}{Z} = \frac{R}{R^2 + \omega_r^2 L^2}$

Hence the impedance of the circuit at resonance

$$Z = \frac{R^2 + \omega_r^2 L^2}{R} = R + \frac{\omega_r^2 L^2}{R}$$

Putting the value of $\omega_r$ from eqⁿ ③ :

$$Z = R + \frac{L^2}{R}\left[\frac{1}{LC} - \frac{R^2}{L^2}\right] = \cancel{R} + \frac{L}{RC} - \cancel{R}$$

$$\boxed{Z = L/RC} \quad — ⑤$$

Parallel resonant circuit is a rejector circuit.

When a complex signal (a mixture of signals of slightly different frequency) is applied across a parallel resonant circuit; the circuit offers maximum impedance to that particular signal whose frequency is same as the resonant frequency of the circuit & corresponding to that particular signal the voltage drop across the circuit is maximum & thus the circuit rejects all other signals. Hence a parallel resonant circuit is regarded as a rejector circuit.

We now show that a parallel resonant circuit gives current amplification.

The r.m.s current flowing in the circuit at resonance

$$I_{min} = \frac{E}{Z_{max}} = \frac{E}{L/RC} \quad (\text{from eq}^n\ ⑤)$$

or $I_{min} \cdot \frac{L}{RC} = E$ — ⑥

The current through the capacitor at resonance

$$I_c = \frac{E}{X_c} = \frac{E}{1/\omega_r C} = E\omega_r C$$

Putting the value of E from eq^n ⑥ & $\omega_r$ from eq^n ③

$$I_c = \frac{I_{min} L}{R\cancel{C}} \cdot \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \cdot \cancel{C}$$

$$\frac{I_c}{I_{min}} = \frac{L}{R}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Since 'R' is the ohmic resistance of the inductance 'R' is generally very small. Hence $\frac{R^2}{L^2}$ can be neglected compared to $\frac{1}{LC}$.

$$\frac{I_c}{I_{min}} = \frac{L}{R}\sqrt{\frac{1}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}} = \text{Q-factor of the circuit.}$$

Since Q-factor is greater than one $I_c > I_{resonance}$.

Similarly $I_L = \frac{E}{X_L} = \frac{E}{\omega_r L}$

putting eq^n ③ and ⑥ :-

$$I_L = I_{resonance} \cdot \frac{L}{RC} \cdot \frac{1}{L\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

$$\frac{I_L}{I_{resonance}} = \frac{1}{RC}\sqrt{LC} = \frac{1}{R}\sqrt{\frac{L}{C}} = \text{Q-factor}$$

Thus at resonance same current flows through the inductive and the capacitive branch & is greater than the resonant current by Q-factor.