We now calculate the expression for the charecteristic length δ from the analysis of electric field screening in a PLASMA.

Suppose that a test charge q is introdu-ced into a plasma, electrons in the plasma will tend to move closer to this charge. Where as +ve ions will tend to move away from it. In a state of statistical equilibrium the spatial distribution of the electrons and ions in the vicinity of the test charge is given by Boltzmann distribution

$$N \propto e^{-\frac{U}{kT}}$$ where $U = P.E$ &

N = no. of particles having P.E U per unit volume.

Let N_e and N_i represent the electron density and the +ve ion density near the test charge.

$$\therefore N_e = N_0 e^{-e\left(\frac{V_0 - V}{KT}\right)}$$

Where V_0 = Plasma potential.

& V = Local potential i.e potential near the test charge.

$$\therefore N_e = N_0 e^{\frac{e(V-V_0)}{KT}} \quad (\text{Charge is -ve})$$

$$\& \quad N_i = N_0 e^{\left[-\frac{e(V-V_0)}{KT}\right]} \quad (\text{charge is +ve})$$

The potential 'V' must satisfy Poisson's eq^n. In spherical polar co-ordinates Poisson's eq^n $\nabla^2 V = \frac{-\rho}{\epsilon_0}$ is written as:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = -\frac{\rho}{\epsilon_0}$$

Assuming the potential to be a function of position co-ordinates 'r' only; being independent of θ & ϕ Poissions equation becomes

$$\frac{1}{r^2}\left[2r\frac{\partial V}{\partial r} + r^2\frac{\partial^2 V}{\partial r^2}\right] = -\left[\frac{N_i e + N_e(-e)}{\epsilon_0}\right]$$

Putting the values of N_i and N_e

$$\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = \frac{N_0 e}{\epsilon_0}\left\{e^{\frac{e(V-V_0)}{KT}} - e^{-\frac{e(V-V_0)}{KT}}\right.$$

or $\frac{d^2V}{dr^2} + \frac{2}{r}\frac{dV}{dr} = \frac{2N_0 e}{\epsilon_0} \sinh\left\{\frac{e(V-V_0)}{KT}\right\}$ — ⑦

Eqⁿ ⑦ is non-linear equation but $KT \gg eV$ (thermal energy is very much high)

i.e $\frac{e(V-V_0)}{KT} \ll 1$

$\therefore \cancel{2}\sinh\left\{\frac{e(V-V_0)}{KT}\right\} \simeq \cancel{2}\frac{e(V-V_0)}{KT}$

Putting in equation ⑦:

$$\frac{d^2V}{dr^2} + \frac{2}{r}\cdot\frac{dV}{dr} = \frac{2N_0 e^2(V-V_0)}{\epsilon_0 KT} \quad \text{— ⑧}$$

Let A = Potential due to the test charge q at a distance r from it.
V is Screening potential = constant.

$A = \frac{q}{4\pi\epsilon_0 r}$ Putting $\frac{\epsilon_0 KT}{2N_0 e^2} = \delta^2$

Eqⁿ ⑧ becomes:

$$\frac{\partial^2 V}{\partial r^2} + \frac{2}{r}\frac{dV}{dr} = \frac{V-V_0}{\delta^2}$$

Put $V - V_0 = x$ (V being function of r, x is also function of r)

$$\therefore \frac{d^2x}{dr^2} + \frac{2}{r}\frac{dx}{dr} - \frac{x}{\delta^2} = 0$$

The solution of the eqⁿ can be written as

$x = V - V_0 = A e^{(-r/\delta)}$

or $V = V_0 + A e^{-r/\delta}$ — ⑨

Applying initial condition it can be shown that the constant $A = \dfrac{q}{4\pi\epsilon_0 r}$

$$\therefore \quad V = V_0 + \frac{q}{4\pi\epsilon_0 r} \exp\left[-\frac{r}{\delta}\right] \quad \text{—— (10)}$$

The constant

$$\delta = \left[\frac{\epsilon_0 KT}{2 n_0 e^2}\right]^{1/2} \quad \text{—— (11)}$$

is Known as Debye's Screening length.