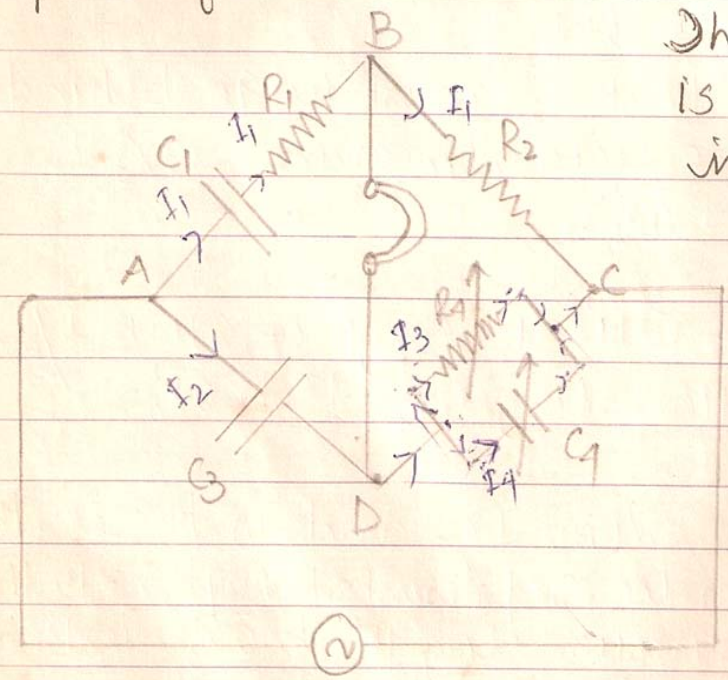
Schering bridge: This bridge is used for the most accurate determination of capacitance of a capacitor ~~and its power factor~~. in terms of a standard capacitor and its power factor.



The circuit connection is made as shown in the diagram.

Let Z_1, Z_2, Z_3 & Z_4 be the impedances of the 1st, 2nd, 3rd and 4th arm respectively.

$$Z_1 = R_1 + \frac{1}{j\omega C_1}, \quad Z_2 = R_2, \quad Z_3 = \frac{1}{j\omega C_3}$$

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$$\frac{1}{Z_4} = \frac{1}{R_4} + \frac{1}{X_c} = \frac{1}{R_4} + \frac{1}{1/j\omega C_4} = \frac{1}{R_4} + j\omega C_4$$

At Equilibrium ; $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$

$$\frac{R_1 + 1/j\omega C_1}{R_2} = \frac{1}{j\omega C_3}\left[\frac{1}{R_4} + j\omega C_4\right]$$

$$\text{Or } \frac{R_1}{R_2} + \frac{1}{j\omega C_1 R_2} = \frac{1}{j\omega C_3 R_4} + \frac{C_4}{C_3}$$

Equating real and Imaginary parts:

$$\frac{R_1}{R_2} = \frac{C_4}{C_3} \quad \text{— (1)}$$

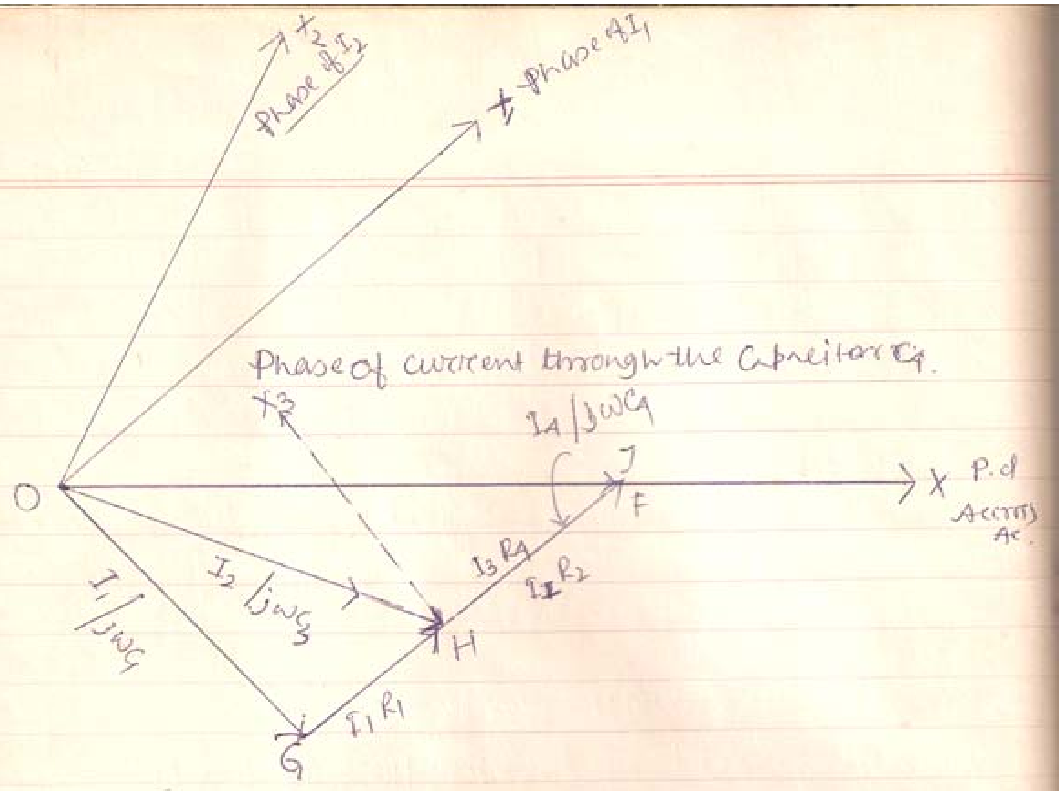
$$\text{or } R_1 = R_2 \cdot \frac{C_4}{C_3} \quad \text{— (1a)}$$

<u>And</u>: $\frac{1}{\cancel{j\omega} C_1 R_2} = \frac{1}{\cancel{j\omega} C_3 R_4}$

$$\text{or } \frac{C_1}{C_3} = \frac{R_4}{R_2} \quad \text{— (2)}$$

$$\text{or } C_1 = \frac{R_4}{R_2} \cdot C_3 \quad \text{— (2a)}$$

We now draw the vector diagram of Schering bridge:



$\overrightarrow{OX}$ represents the phase of the P.D V_{AC}

$\boxed{\overrightarrow{OF} = \overrightarrow{V_{AC}}}$

$\overrightarrow{OX_1}$ and $\overrightarrow{OX_2}$ represent the phase of the current I_1 & I_2 through the branch AB and AD respectively.

$V_{AB} = V_{C_1} + V_{R_1} = I_1 \cdot \frac{1}{j\omega C_1} + I_1 R_1$

Hence V_{C_1} must lag behind I_1 by 90° and is represented by $\overrightarrow{OG}$. $\overrightarrow{OG} = I_1/j\omega C_1$

$V_{R_1} = I_1 R_1$ being in phase with current I_1 can be represented by a vector $\overrightarrow{GH} = I_1 R_1$ along the direction of $\overrightarrow{OX_1}$.

$\therefore \quad \overrightarrow{OH} = \overrightarrow{OG} + \overrightarrow{GH} = \overrightarrow{V_{C_1}} + \overrightarrow{V_{R_1}} = \overrightarrow{V_{AB}}$

But $\overrightarrow{V_{AB}} = \overrightarrow{V_{AD}}$

$\therefore \quad \overrightarrow{OH} = \overrightarrow{V_{AD}} = I_2 \cdot \frac{1}{j\omega C_3}$ hence $\overrightarrow{OH}$ must be 90° behind $\overrightarrow{OX_2}$ phase of current I_2.

The P.D across R_2, $V_{BC} = I_1 R_2$ and this P.D being in phase with current I_1 can be represented by a vector $\overrightarrow{HJ}$ of length $I_1 R_2$, along the direction of $\overrightarrow{OX_1}$.

$\therefore \quad \overrightarrow{V_{AB}} + \overrightarrow{V_{BC}} = \overrightarrow{OH} + \overrightarrow{HJ} = \overrightarrow{OJ} = \overrightarrow{V_{AC}} = \overrightarrow{OF}$

Hence 'J' and 'F' must be the same point.

The P.D across DC:

$$\overrightarrow{V_{DC}} = \overrightarrow{I_3} R_4 = \overrightarrow{I_4} \cdot \frac{1}{j\omega C_4}$$

where I_3 and I_4 must be the current through the resistance R_4 and capacitance C_4 respectively.

Since $\quad V_{BC} = V_{DC}$ & $\overrightarrow{V_{BC}} = \overrightarrow{I_1} R_2$

$\overrightarrow{V_{DC}} = \overrightarrow{I_3} R_4$

Hence phase of current I_3 must be same as that of I_1.

Hence $I_3 R_4$ is represented by vector $\overrightarrow{HJ}$ along the direction of OX_1.

I_4 leads the P.D V_{DC} by 90°.

Hence phase of current I_4 must be chosen along $\overrightarrow{HX_3}$ perpendicular to $\overrightarrow{HJ}$.

$\therefore$ P.D across C_4, $\overrightarrow{V_{C_4}} = \frac{I_4}{j\omega C_4} = \overrightarrow{HJ}$ of length $\frac{I_4}{\omega C_4}$ & the direction along perpendicular to $\overrightarrow{HX_3}$.