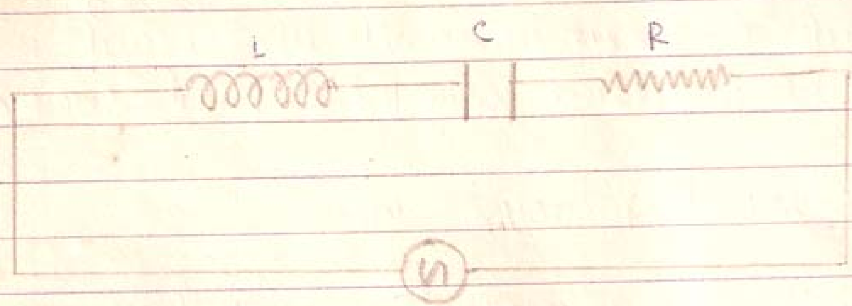
Series Resonant Circuit:



We Know that when an alternating Source of e.m.f is connected across an L-C-R Series circuit; the circuit may behave as an inductive circuit or a capacitative circuit or a purely resistive circuit, depending on the values of ω, L & C.
It is found that when $\omega = \frac{1}{\sqrt{LC}}$, the circuit behaves as a purely resistive circuit and the current flowing in the circuit is maximum.
We Know that an L-C-R circuit is an oscillating circuit the freq. of oscillation is given by

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

Since R in the circuit is generally very small $\frac{R^2}{4L^2}$ can be neglected compared to $\frac{1}{LC}$ & hence the frequency of oscillatory circuit

$$\omega \simeq \frac{1}{\sqrt{LC}}$$

Thus we find that when an alternating source connected across a series L-C-R circuit, has a frequency same as the frequency of that L-C-R circuit; the current (r.m.s) flowing in the circuit is maximum and the circuit is said to be in resonance with the applied source of e.m.f.

Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$

If the frequency of the applied source is too high $X_L = \omega L >> X_C = 1/\omega C$ & the P.D across the circuit leads the current by a phase angle, nearly equal to $\pi/2$ and the circuit is inductive.

For too low frequency $X_L = \omega L << X_C = \frac{1}{\omega C}$ & the P.D across the circuit lags the current by phase angle nearly $\pi/2$ and the circuit is capacitive circuit.

But for middle range frequency of the source if $\omega L = \frac{1}{\omega C}$ i.e $X_L = X_C$, the P.D across the inductance $V_L = I_{r.m.s} X_L$ and across the capacitor $V_C = I_{r.m.s} X_C$ are equal in magnitude and 180° out of phase, will mutually cancel out. Hence the e.m.f of the applied source has to overcome the P.D across the resistance only $V_R = I_{rms} R$. & hence the (r.m.s) current in the circuit becomes max^m.

The circuit is therefore a Resonant circuit.

We now show that series resonant circuit produces a voltage amplification. We know that in series L-C-R circuit (r.m.s) current is given by $I = \frac{E}{Z}$ where E = applied e.m.f or $I = \dfrac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$ —— ①

when $\omega = \omega_r = \dfrac{1}{\sqrt{LC}}$ i.e at resonance $\omega_r L = \dfrac{1}{\omega_r C}$

and I is maximum

from ①;

$$I_{max} = \frac{E}{R} \quad \text{—— ②}$$

At resonance the P.D across the inductance

$$V_L = I_{max} X_L = \frac{E}{R} X_L$$

or $\dfrac{V_L}{E} = \dfrac{X_L}{R} = \dfrac{\omega_r L}{R}$ = Q factor or figure of merit

Since $\omega_r L \gg R$, $V_L \gg E \Rightarrow$ The P.D across the inductance at resonance is much greater than the applied e.m.f indicating a voltage amplification.

The P.D across the capacitance at resonance

$$V_C = I_{max} X_C = \frac{E}{R} \frac{1}{\omega_r C}$$

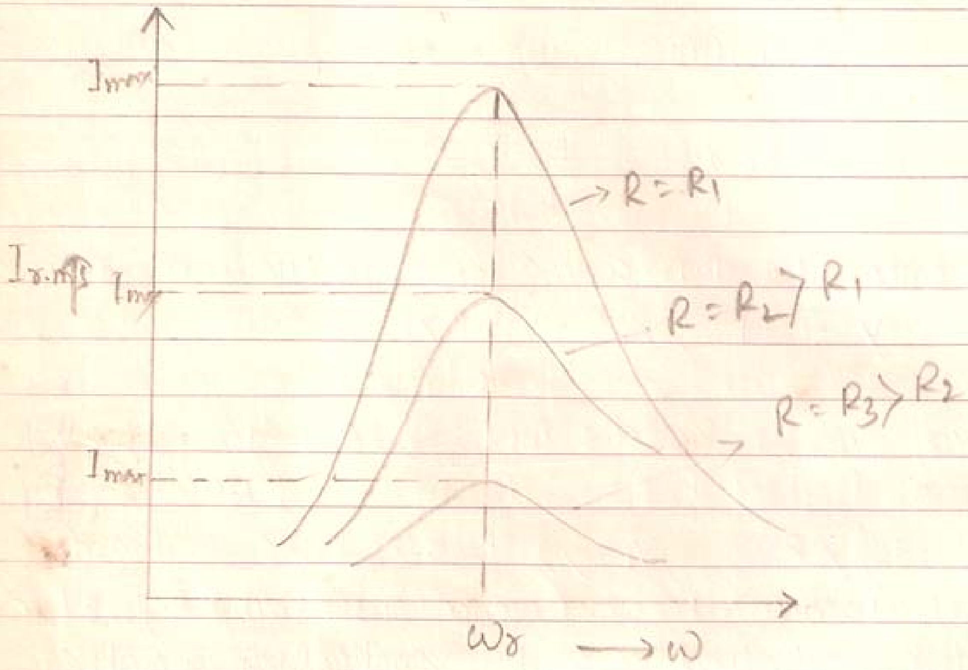
or $\dfrac{V_C}{E} = \dfrac{1}{R \omega_r C}$

This ratio of P.D across the inductance or capacitance to the applied e.m.f at resonance is known as Q-factor or figure of merit.

$$\text{Q-factor} = \frac{V_L}{E} = \frac{1/\sqrt{LC} \cdot L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{V_C}{E} = \frac{1}{R} \frac{1}{\frac{1}{\sqrt{LC}} \cdot C} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

To have a high Q- factor:
(i) Resistance in the circuit should be Sm
(ii) Inductance should be high.
(iii) Capacitance should also be small.



$I_{max} = \frac{E}{R}$

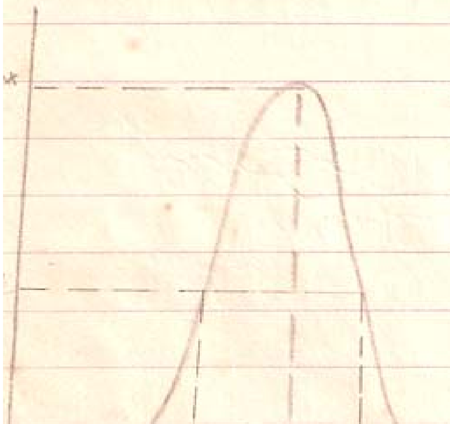
The three curves are for three circuits with different values of R. From the graph we find that as the resistance in the circuit increases the maximum value of current at resonance decreases.

A Series resonance circuit is known as an acceptor circuit. When a signal of slightly different frequencies appear simultaneously across the circuit; the circuit shows max^m current only corresponding to the signal with which it is in resonance.

Hence the circuit accepts that particular signal whose frequency is same as the resonant frequency of the circuit & hence it is known as an acceptor circuit.

Sharpness of Resonance:

We find that as the frequency of the applied e.m.f varies from the resonant value the current (r.m.s) in the circuit decreases from the peak value. Sharpness of resonance measures how rapidly the current (r.m.s) falls from its peak value as the frequency decreases or increases from its resonant value. The frequency of the applied e.m.f at which the current (r.m.s) in the circuit falls to $\frac{I_{max}}{\sqrt{2}}$ is known as half power frequency. The gap between two half power frequencies is a measure of the sharpness of resonance. Smaller is the gap sharper is the curve.



We now find the sharpness:

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

When $\omega = \omega_r$; $\omega_r L = \frac{1}{\omega_r C}$

& $I = I_{max} = \frac{E}{R}$

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{or } I = \frac{E}{R} \cdot \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{I_{max}}{\sqrt{R^2 + (}}$$

$$\therefore I = \frac{I_{max} R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

When $\omega = \omega_h$

$$I = \frac{I_{max}}{\sqrt{2}}$$

$$\therefore \frac{\cancel{I_{max}}}{\sqrt{2}} = \frac{\cancel{I_{max}} R}{\sqrt{R^2 + \left(\omega_h L - \frac{1}{\omega_h C}\right)^2}}$$

Squaring both Sides and re-arranging:

$$R^2 + \left(\omega_h L - \frac{1}{\omega_h C}\right)^2 = 2R^2$$

$$\text{or} \quad \left(\omega_h L - \frac{1}{\omega_h C}\right)^2 = R^2$$

$$\text{or} \quad \omega_h L - \frac{1}{\omega_h C} = \pm R$$

Let ω_1 & ω_2 be the two values of ω_h

$$\omega_2 L - \frac{1}{\omega_2 C} = +R \quad \text{—} \quad (X)$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{—} \quad (Y)$$

Adding eq^n (X) and (Y):

$$(\omega_1 + \omega_2) L - \frac{1}{C}\left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) = 0$$

$$\boxed{\omega_1 \omega_2 = \frac{1}{LC}}$$

Substracting eq^n (Y) from (X)

$$(\omega_2 - \omega_1) L + \frac{1}{C}\left(\frac{\omega_2 - \omega_1}{\omega_1 \omega_2}\right) = 2R$$

$$\text{Or} \quad (\omega_2 - \omega_1)\left[L + \frac{1}{\cancel{C}} \cdot L\cancel{C}\right] = 2R$$

or $(\omega_2 - \omega_1) \cdot \cancel{2}L = \cancel{2}R$

Or $\boxed{(\omega_2 - \omega_1) = \frac{R}{L}}$

Sharpness of resonance is defined as the ratio of Resonant frequency to the difference of half power frequency.

i.e Sharpness of Resonance $= \frac{f_r}{f_2 - f_1} = \frac{\omega_r}{\omega_2 - \omega_1}$

$$= \frac{1/\sqrt{LC}}{R/L} = \frac{L}{R}\frac{1}{\sqrt{LC}}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} = \underline{Q\text{-factor.}}$$ Thus

Sharpness of resonance is decided by the Q-factor.