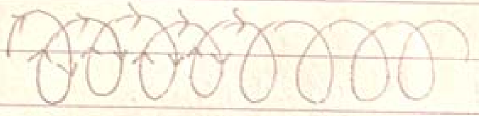
Current Electricity:

Magnetic field due to a Solenoidal conductor:



When a conductor is kept in the form of a Spiral coil, it is known as Solenoid.

Given: N = Total no. of turns in the Solenoid.

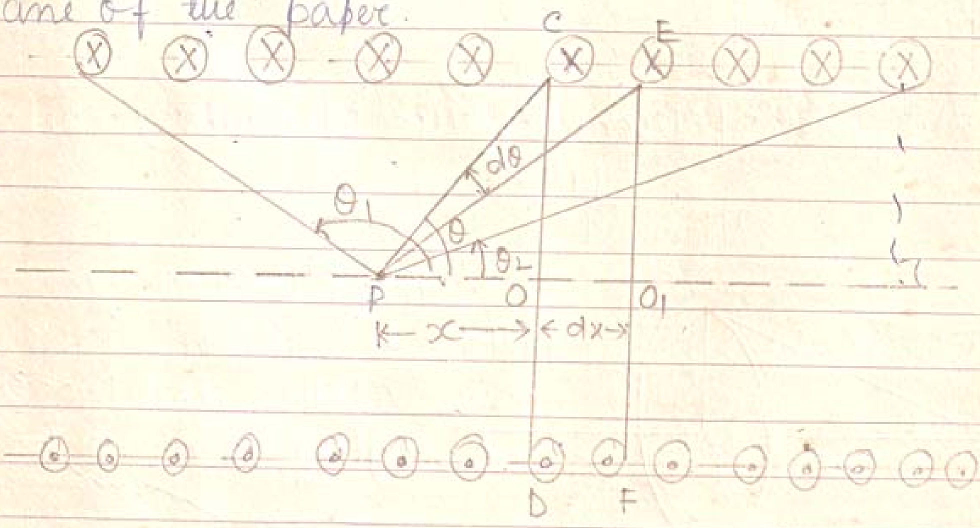
a = Radius of the coil.

i = current flowing through the Solenoid.

The no. of turns per unit length of the Solenoid

$$= \frac{N}{l}$$

Let us show the Section of the Solenoid by the plane of the paper.



The crosses ⊗ represent the Sections of the conductors carrying current into the plane of the paper; away from the observer.

The dots ⊙ represent the Sections of the Conductor carrying current out of the plane of the paper towards the observer.

Let 'P' be the point on the axis of the Solenoid; where induction vector of the magnetic field due to the current in the Solenoid is to be calculated.

Let us choose the point 'P' as origin.

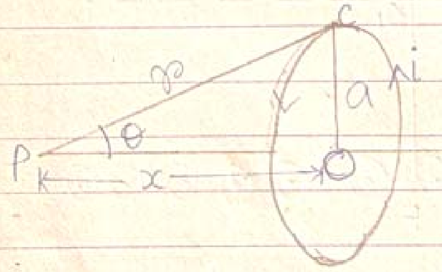
θ_1 & θ_2 = Angles subtended by the left and right ends of the solenoid with the axis of the solenoid at the point P.

Let us consider an elementary section of length 'dx' at a distance 'x' from the origin of the solenoid, by two planes perpendicular to the axis of the solenoid. 'CD' and 'EF' are sections of the two planes by the plane of the paper. This elementary section appears like a circular ring. Join PC and PE. Let $O\hat{P}C = \theta$; $E\hat{P}C = d\theta$

Let $\boxed{PC = r}$

The number of turns in the elementary coil

$$n = \frac{N}{L} dx$$



We know that the magnetic field at a point on the axis of the circular coil carrying a current

$$B = \frac{\mu_0}{2\cdot 4\pi} \frac{2\pi\, n i a \sin\theta}{r^2} \quad \text{--- (1)}$$

$$\frac{\mu_0}{4\pi} \frac{2\pi n i a^2}{(a^2+x^2)^{3/2}}$$

Using eq^n (1) the induction vector at the point P due to the elementary circular coil carrying current

$$dB = \frac{\mu_0}{2} \frac{a N}{L} \frac{dx \; i \sin\theta}{r^2} \quad \text{--- (2)}$$

The direction of this induction vector is along the axis of the coil, such that if we curl the fingers of right hand in the direction of current the thumb gives the direction of induction vector.

If we consider the elementary circular coil, on either side of the point P, the direction of the induction vector remains same. Hence the vector addition reduces to scalar addition and the resultant magnetic induction vector at P, due to the solenoid can be obtained by integrating eq^n (2).

Eq^n (2) contains three variables, r, θ & x hence can't be integrated as it is

$$\tan\theta = \frac{a}{x} \quad \text{or} \quad x = a\cot\theta, \; \boxed{dx = -a\,\mathrm{cosec}^2\theta\, d\theta}$$

$$\mathrm{Cosec}\,\theta = \frac{r}{a} \; ; \; r = a\,\mathrm{cosec}\,\theta$$

$$\therefore B = \int dB = \int_{\theta_1}^{\theta_2} \frac{\mu_0 a}{2} \frac{N i}{L} \frac{(-\cancel{a}\,\cancel{\mathrm{cosec}^2\theta}\, d\theta)\sin\theta}{\cancel{a^2}\,\cancel{\mathrm{cosec}^2\theta}}$$

$$\text{or } B = -\frac{\mu_0}{2} \frac{N i}{L} \frac{1}{\cancel{a}} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\text{or } B = \frac{\mu_0}{2} \frac{N i}{L\cancel{a}} [\cos\theta_2 - \cos\theta_1] \quad \text{--- (3)}$$

If the length of the Solenoid is large compared to its diameter and the point is well inside the Solenoid, then

$\theta_2 = 0^\circ$ & $\theta_1 = \pi$

from (3):-

$$B = \frac{\mu_0}{2}\frac{Ni}{La}\left[\cos 0 - \cos\pi\right]$$

$$= \frac{\mu_0}{2}\frac{Ni}{La}\left[1 - (-1)\right]$$

$$B = \frac{\mu_0}{2}\frac{Ni}{La} \times 2$$

$$\boxed{B = \frac{\mu_0 Ni}{La}} \quad \text{--- (4)}$$

$B = \frac{\mu_0 Ni}{L}$ Newton/am