Law of EQUIPARTITION OF ENERGY:

Statement: " The total K.E of a dynamic system consisting of large no. of particles in thermal equilibrium is equally divided among all of its degrees of freedom and the average energy associated with each degree of freedom is $\frac{1}{2}$ KT; whe k is Boltzmann constant and T is the absolute temp. of the System."

Proof: Consider a dynamical System with f-degree of freedom. Classically the system is described by f position co-ordinates $q_1, q_2 \cdots q_f$ and the corresponding f momenta co-ordinates $p_1, p_2 \cdots p_f$. The total energy of the System is given by

$$E = E(q_1, q_2 \cdots q_f; p_1, p_2, \cdots p_f) \quad -(1)$$

Let p_i be any particular momentum then eq^n (1) becomes:

$$E = \{\epsilon_i(p_i) + E'(q_1 \cdots q_f; p_1 \cdots p_f)\} \quad -(2)$$

Where,

$\epsilon_i(p_i)$ is the function of momentum p_i alone and the Second term $E'(q_1 \cdots q_f; p_1 \cdots$ is the function of all the position and momenta co-ordinate excluding the momentum p_i.

Now,

The mean energy $(\bar{\epsilon}_i)$ is defined as

$$\bar{\epsilon}_i = \frac{E}{N} = \left\{ \frac{\int \epsilon_i e^{-E/kT} \cdot dq_1 \cdot dq_2 \cdots dq_f \cdot dp_1 \cdots dp_f}{\int e^{-E/kT} dq_1 \cdot dq_2 \cdots dq_f \cdot dp_1 \cdots dp_f} \right\}$$

Now,

$$E = (\epsilon_i + E')$$

So that eq^n (3) becomes

$$\bar{\epsilon}_i = \left\{ \frac{\int \epsilon_i e^{-(\epsilon_i + E')/kT} dq_1 \cdots dq_f \cdot dp_1 \cdots dp_f}{\int e^{-(\epsilon_i + E')/kT} dq_1 \cdots dq_f \cdot dp_1 \cdots dp_f} \right\}$$

$$= \left[\left\{ \frac{\int_{-\infty}^{+\infty} \epsilon_i e^{-\epsilon_i/KT} dp_i}{\int_{-\infty}^{+\infty} e^{-\epsilon_i/KT} dp_i} \right\} \left\{ \frac{\int_{-\infty}^{+\infty} e^{E'/KT} \cdot dq_1 \cdots dq_f \, dp_1 \cdots dp_f}{\int_{-\infty}^{+\infty} e^{E'/KT} dq_1 \cdots dq_f \, dp_1 \cdots dp_f} \right\} \right]$$

$$\therefore \overline{\epsilon_i} = \frac{\int_{-\infty}^{+\infty} \epsilon_i e^{-\epsilon_i/KT} dp_i}{\int_{-\infty}^{+\infty} e^{-\epsilon_i/KT} dp_i}$$

Again, $\epsilon_i = \frac{p_i^2}{2m}$ ——— (5)

where m = mass of the particle.

From eq^n (4) and (5) we have,

$$\overline{\epsilon_i} = \left\{ \frac{\int_{-\infty}^{+\infty} \frac{p_i^2}{2m} e^{-p_i^2/2mKT} dp_i}{\int_{-\infty}^{+\infty} e^{p_i^2/2mKT} dp_i} \right\}$$

$$= \frac{\frac{1}{2m} \int_{-\infty}^{+\infty} p_i^2 e^{-\frac{1}{2mKT} \cdot p_i^2} \cdot dp_i}{\int_{-\infty}^{+\infty} e^{-\frac{1}{2mKT} \cdot p_i^2} \cdot dp_i}$$

$$= \frac{\frac{1}{2m} \left[\frac{1}{2} \sqrt{\frac{\pi}{(1/2mKT)^3}} \right]}{\sqrt{\frac{\pi}{(1/2mKT)}}}$$

$$= \frac{1}{2m} \cdot \frac{1}{2} \sqrt{\frac{(\pi/2mKT)}{(\pi/2mKT)}} \cdot (2mKT)$$

Hence, $\boxed{\overline{\epsilon_i} = \frac{1}{2} KT}$