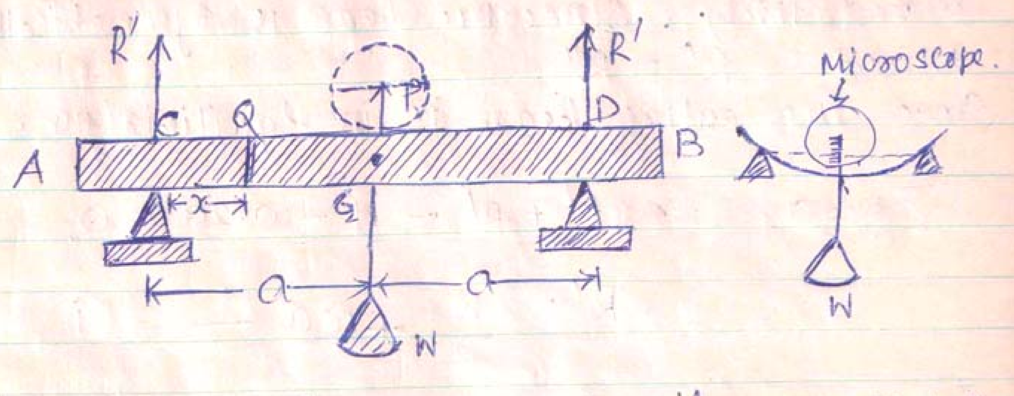
Determination of Youngs modulus by ~~NON~~ - NON UNIFORM BENDING OF BEAM:



Description: AB is a beam of uniform cross-section throughout; and is supported uniformly on the knife edges C & D. So that the length of the portions AC and BD are very small and equal. G is the mid-point of the beam where a hanger is suspended and pointer P is also attached on the beam at G. When load is put on the hanger; the beam is bent, the pointer P comes down and using a travelling microscope the depression at the mid-point is measured.

Theory:
Let $2a$ = length of the beam AB.
w = weight per unit length of the beam.
W = the load suspended at the mid point G.
The weight of the portions AC & BD are neglected and hence they should be kept equal and very small. Since the length of the portions AC and BD are very small; $\boxed{AB \simeq CD = 2a}$ $\therefore$ $\boxed{CG = GD = a}$

Let R' & R'' be the normal reaction of the Knife edges on the beam, at the points C and D respectively. Since the beam is symmetrically placed $R' = R''$

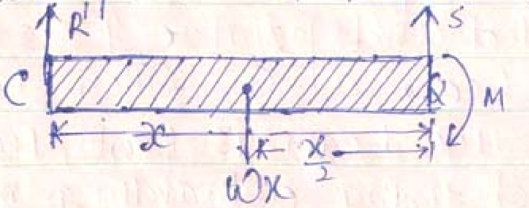
Since the entire beam is in Equilibrium.

$$\sum F = 0 \quad \text{or} \quad R'' + R' - W - \omega \cdot 2a = 0$$

$$R' = \frac{W}{2} + \omega a \longrightarrow (1)$$

Let us Consider any point $Q(x, y)$ at a distance x from the Knife edge C which is taken as Origin. Imagine a section of the beam, at Q. The portion CQ is shown separately below.

Since the portion CQ is in Equilibrium



$$\sum F = R' + S' - \omega x = 0$$

$$S' = \omega x - R'$$

$$= \omega x - \frac{W}{2} - \omega a$$

Taking moment about the point Q:

$$R'x + M - \omega x \cdot \frac{x}{2} = 0$$

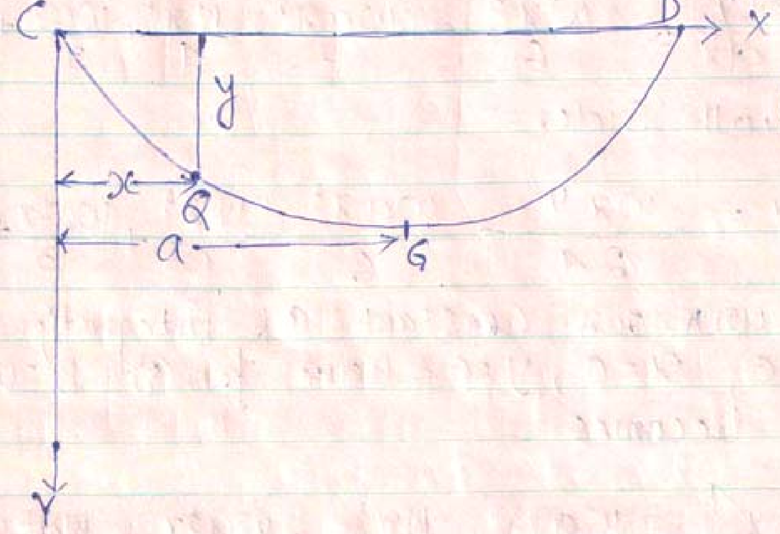
$$M = \frac{\omega x^2}{2} - R'x$$

Putting Equation (1): $M = \frac{\omega x^2}{2} - \frac{Wx}{2} - \omega a x$

Let 'R' be the Radius of curvature of bend at the point Q:

$$M = \frac{YAK^2}{R} = \frac{\omega x^2}{2} - \frac{Wx}{2} - \omega a x \longrightarrow (2)$$

From Equation ② we find that Radius of curvature 'R' depends on x, and hence is different at different points in the beam. Hence the beam is said to be Non-Uniformly bent.



(magnified diagram)

From Calculus the curvature at Q; assuming the curvature to be small

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Putting in Equation ②:-

$$YAk^2 \frac{d^2y}{dx^2} = \frac{wx^2}{2} - wax - \frac{Wx}{2} \longrightarrow ③$$

Equation ③ is a 2nd Order differential Equation.
Integrating both sides:-

$$YAk^2 \frac{dy}{dx} = \frac{wx^3}{6} - \frac{wax^2}{2} - \frac{Wx^2}{4} + c' \longrightarrow ④$$

Where c' is an unknown Constant of Integration.
At the mid-point G; the tangent is ll^{el} to the x-axis $\psi = 0$; $\tan\psi = \frac{dy}{dx} = 0$ & $x = a$
from ④:-

$$0 = \frac{wa^3}{6} - \frac{wa^3}{2} - \frac{Wa^2}{4} + c'$$

or, $C' = \frac{wa^3}{3} + \frac{Wa^2}{4} \longrightarrow$ (5)

Putting this value of C' in Equation (4) :

$$YAK^2 \frac{dy}{dx} = \frac{wx^3}{6} - \frac{wax^2}{2} - \frac{Wx^2}{4} + \frac{wa^3}{3} + \frac{W}{4}a^2$$

Integrating both sides :-

$$YAK^2 y = \frac{wx^4}{24} - \frac{wax^3}{6} - \frac{Wx^3}{12} + \frac{wa^3 x}{3} + \frac{W a^2 x}{4}$$

Where D' is unknown Constant of integration
At the origin C, $x=0$, $y=0$ & hence from (5) $D'=0$
Equation (5) becomes

$$YAK^2 y = \frac{wx^4}{24} - \frac{wax^3}{6} - \frac{Wx^3}{12} + \frac{wa^3 x}{3} + \frac{Wa^2 x}{4} \longrightarrow (6)$$

let 'δ' be the depression at the mid-point G
at G ; $\boxed{x = a}$ & $\boxed{y = \delta}$

from (6) :- $YAK^2 \delta = \frac{wa^4}{24} - \frac{wa^4}{6} - \frac{Wa^3}{12} + \frac{wa^4}{3} + \frac{W a^3}{4}$

$\therefore YAK^2 \delta = \frac{5}{24} wa^4 + \frac{1}{6} Wa^3 \longrightarrow$ (7)

From Equation (7) we find that ; the depression is the sum of the two terms (i) the first term arises due to the weight of the beam (ii) the second term arises due to the external Load applied at the mid-point.

Let W_0 = Weight of the hanger only which suspended at G. $[W = W_0 + mg]$

Let "δ_0" be the depression at the mid point G when no load 'mg' is put on the hanger ie the depression due to the weight of the beam and weight of the hanger only.

from Equation ⑦:-

$$YAk^2\delta_0 = \frac{5}{24}wa^4 + \frac{1}{6}W_0a^3 \longrightarrow (8)$$

$\frac{a^2x}{4} + D' \longrightarrow (5)$

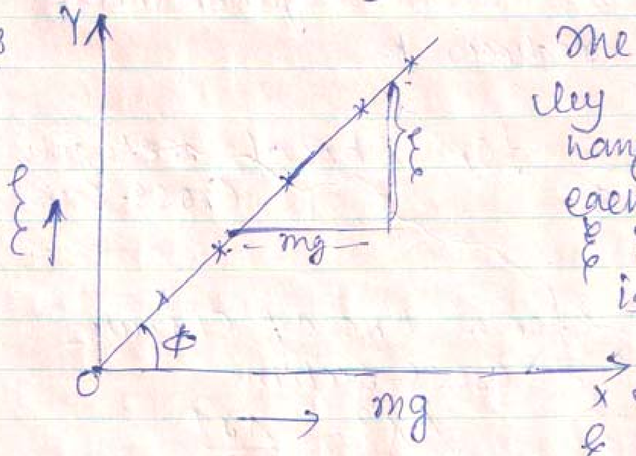
Substracting Equation ⑧ from ⑦:-

$$YAk^2(\delta - \delta_0) = \frac{(W-W_0)}{6}a^3 = \frac{mga^3}{6}$$

Let $\delta - \delta_0 = \xi$ = The depression at the mid-point G due to the load 'mg' put on the hanger

$$\therefore YAk^2\xi = \frac{1}{6}mga^3 \text{ - - - - - - } (9)$$

$\frac{a^3}{4}$



The experiment is repeated by increasing load on the hanger in step and for each load the corresponding ξ is measured. A graph is plotted which is found to be a straight line as shown.

The slope $\tan\phi = \frac{\xi}{mg}$ - - - (10)

Let l = length of the bar = $2a$ · or $\boxed{a = \frac{l}{2}}$

b & d = breadth and depth of the bar ie the lengths of the horizontal and vertical sides of the cross-section (rectan

$$Ak^2 = \frac{bd^3}{12}$$

from Equation ⑨ :-

$$Y = \frac{1}{6} \frac{(l/2)^3}{\frac{bd^3}{12}} \cdot \frac{mg}{\xi}$$

$$\text{or } Y = \frac{l^3}{4bd^3} \cdot \frac{1}{\tan\phi} \longrightarrow ⑪$$

$\because \frac{mg}{\xi} = \frac{1}{\tan\phi}$ from ⑩

Using Equation ⑪ ; Youngs modulus can be Calcu

—x—

For numerical: δ_0 can be neglected hence the above formula reduces to :-

$$Y = \frac{Wl^3}{4bd^3\delta} \quad \text{for a bar of rectangular cross-section}$$

& $YAk^2\delta = \frac{Wa^3}{6}$

$$Y = \frac{Wa^3}{6Ak^2\delta} \quad \text{for a bar of circular cross-section}$$

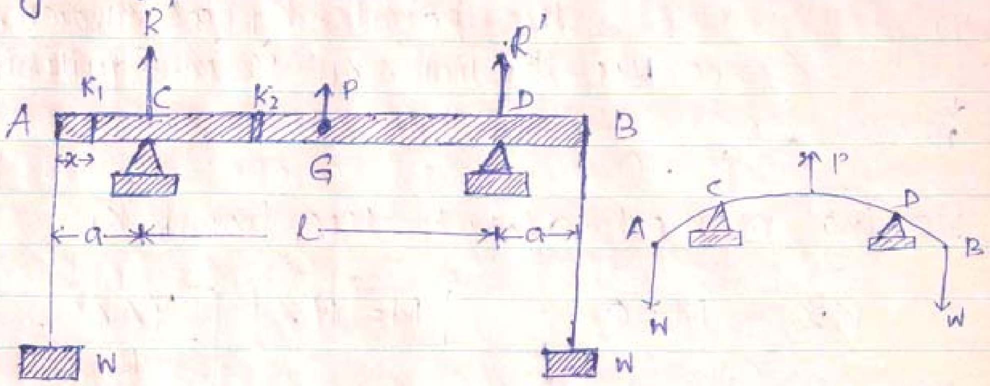
$$Ak^2 = \frac{\pi r^4}{4}$$

$$a = \frac{l}{2}$$

$$Y = \frac{Wl^3}{2^3 \times 6 \times \frac{\pi r^4}{4}\delta}$$

$$\boxed{Y = \frac{Wl^3}{12\pi r^4\delta}}$$

Determination of Youngs modulus by uniform bending of beam.



Description: Let us consider a very light beam (the weight of the beam is negligible) of uniform cross-section throughout its entire length. The beam is supported symmetrically on the two knife edges C and D so that the length of the portions AC and BD are equal. Two equal weights are suspended at the two free ends A and B, the beam is bent up; the pointer P at the middle point G rises up. Using a travelling microscope focussed at P; the rise in height can be measured.

Theory: $\boxed{AC = BD = a}$ $\boxed{CD = l}$

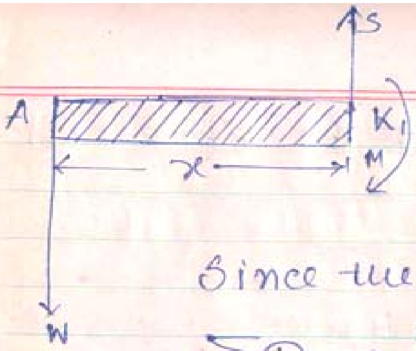
W = The load put at each end A and B

let R' be the normal reaction of the knife edges on the beam; at the points C and D & due to symmetry the reactions at C and D are same.

Since the beam is in Equilibrium;

$\Sigma F = 0$ ie $R' + R' - W - W = 0$ $R' = W \longrightarrow (1)$

Imagine a section of the beam at K_1 between A and C, at a distance x from the end A. The portion 'AK_1' is shown separately magnified



let S and M be the Shearing force and B.M at the point K_1 as shown.

Since the portion AK_1 is in Equilibirium

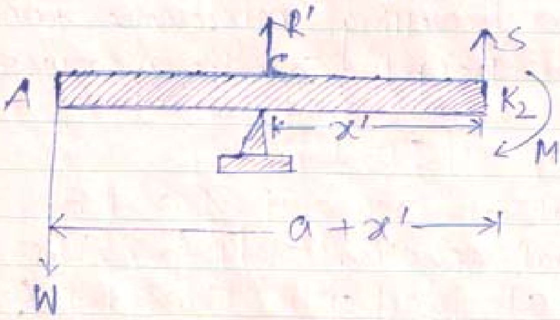
$\sum T = 0$

Taking moment about the point K_1

$Wx - M = 0$ $\boxed{M = Wx}$ $\boxed{\frac{YAK^2}{R} = Wx} \rightarrow$ (2)

Where R is the radius of curvature of bending at K_1 from Equation (2) we find that R depends on x. Thus for any point between A and C, the radius of curvature depends on the distance of the point from the load. Hence the portion AC is non-uniformly bent.

let us Consider a point K_2 between A and C at a distance x' from the Knife edge C.



let S and M be the Sheareing force and B.M at K_2.

Since the portion AK_2 is in Equilibirium

$\sum T = 0$ Taking moment about K_2

$$-W(a+x') + R'x' + M = 0$$

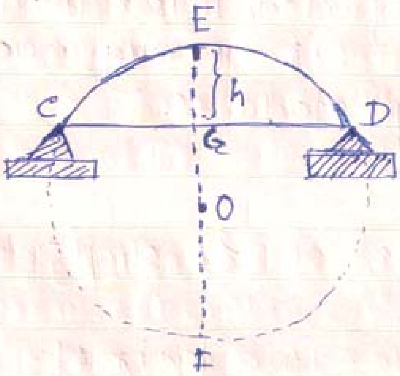
$$M = W(a+x') - R'x'$$

$$= Wa + \cancel{Wx'} - \cancel{Wx'}$$

$$= Wa$$

$$\therefore \boxed{M = \frac{YAK^2}{R} = Wa} \longrightarrow (3)$$

Where R is the radius of curvature of bending at the point K_2.
from Equation ③ we find that the radius of curvature is independent of the distance of the point K_2 & hence R is const for every point between C & D. Hence the portion of the beam between the knife edges C & D is uniformly bent; while the portions AC & BD outside the knife edges are non uniformly bent.



The portion CD of the bar which is uniformly bent is shown in the form of an arc of a circle with centre 'O' & Radius of curvature R.

let GE = h the height through which the middle point G rises up when the beam is bent up as measured by the travelling microscope.
From geometry: $CG \cdot GD = EG \cdot GF$

$$\frac{l}{2} \cdot \frac{l}{2} = (2R-h) \cdot h = 2Rh - h^2$$

h being small; h^2 can be neglected compared to 2Rh.

$$\therefore \frac{l^2}{4} = 2Rh \quad \text{or} \quad R = \frac{l^2}{8h} \longrightarrow ④$$

Putting Equation ④ in ③ :-

$$YAk^2 \frac{8h}{l^2} = Wa$$

$$Y = \frac{Wal^2}{8Ak^2 h} \longrightarrow ⑤$$

For a bar of ractangular cross-section ;

$$Ak^2 = \frac{bd^3}{12}$$

Where b and d are the lengths of the horizontal and vertical sides of the cross-section.

$$\therefore Y = \frac{3}{2} \frac{Wal^2}{bd^3 h} \longrightarrow (6)$$

Remark : Since h^2 is being neglected this method of uniform bending of beam is not very accurate for determination of Young's modulus.