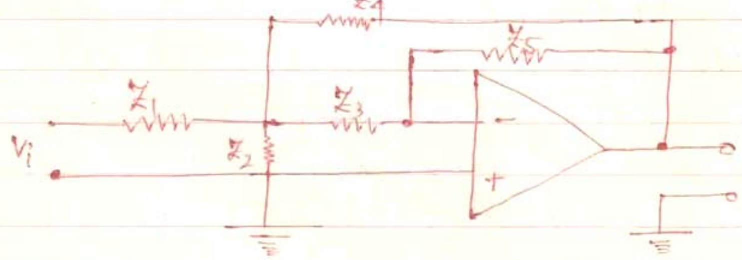
Butter worth filter :

Qr. What is an active filter? Explain the necessary theory of the low-pass-filter of the 2ND order. Hence give the design characteristic of active-low pass filter of fourth order.

ANS : Active filter :

Active circuit filters were developed through the simulation of LC-filter-circuits by analogue computer technique. Combination of R & C elements of the OP-amp which is active element provide the filter simulation. The OP-AMP as the gain element is preferred over the LC filter form for low frequencies:



Active filter.
FIG: 1.

The circuit of an active filter is given in fig 1. It is possible to design to active filters up to frequencies of several MHz, since

Commercially available OP-AMP's have unity gain band width.

Theory of the active low-pass-filter of the Second order (Butterworth filter):

An approximation for an ideal low-pass-filter (because the ideal filter is unrealizable with physical elements) of the form.

$$A_v(s) = \frac{1}{P_n(s)} \quad \text{——} \; (1).$$

where,

$P_n(s)$ is a Polynomial in the variable s.

Further, for Butterworth filter, a common approximation of eq^n (1) using the "Butterworth-polynomials" $B_n(s)$ we have,

$$A_v(s) = \frac{A_{v_0}}{B_n(s)} \quad \text{——} \; (2).$$

and with, $s = j\omega$

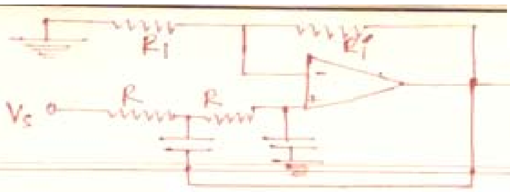
$$|A_v(s)|^2 = |A_v(s)|\,|A_v(-s)|$$

$$|A_v(s)|^2 = \left[\frac{A_{v_0}^2}{1+(\omega/\omega_0)^{2n}}\right] \text{——} \; (3).$$

from eq^n (2) and (3); the magnitude of $B_n(\omega)$ is given by,

$$|B_n(\omega)| = \left\{\sqrt{1+(\omega/\omega_0)^{2n}}\right\} - (4).$$

The larger the value of n more closely the case approximates to the ideal low-pass-filter response (Graph shown below)

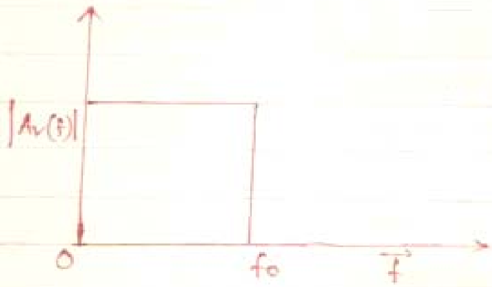


Second order low pass filter.

FIG : 2

A table containing normalized Butterworth polynomials is available which shows that

(i) For even n, the polynomials are ~~present~~ product of quadratic form and



ideal Characteristic low-pass filter.

(ii) For odd n, the additional factor (s+1) is present. The zeros of the normalized Butterworth polynomials are either ⊥ or complex conjugate & are found only on the so called Butterworth circle of unit radius.

Now, from the table and eq^n ②, we have the typical second order Butterworth filter "transfer-function" of the form given below:-

$$\frac{A_v(s)}{A_{v_0}} = \left[\frac{1}{(s/\omega_0)^2 + 2k(s/\omega_0) + 1} \right] \quad \text{—⑤}$$

where,

K is the damping factor.

& $\omega_0 = 2\pi f_0$

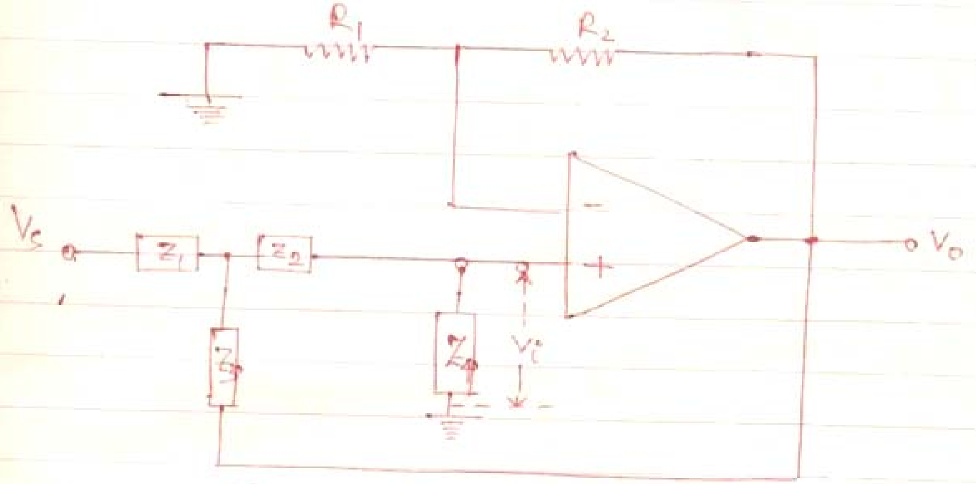
Similarly for the 1st order filter we have

$$\frac{A_v(s)}{A_{v_0}} = \left\{ \frac{1}{(s/\omega_0) + 1} \right\} \quad \text{— ⑥}$$

__Practical Circuit:__

The generalized active filter circuit is shown in

FIG: 3, where the active element is an OP-AMP.



Generalised active filter. FIG: 3.

Now, in order to determine the stable mid-band gain,

$$\frac{V_o}{V_i} = A_{vo} = \frac{(R_1 + R_1')}{R_1}$$

We assume that amplifier input current is zero. Further we have the following relation

$$A_v(s) = \frac{V_o}{V_s} = \left\{ \frac{A_{vo} Z_3 Z_4}{Z_3(Z_1 + Z_2 + Z_3) + Z_1 Z_2 + Z_1 Z_4 (1 - A_{vo})} \right\} \quad (1)$$

If this network is to be a low pass filter then Z_1 & Z_2 are resistances and Z_3, Z_4 are capacitances.

Assuming:

$$Z_1 = Z_2 = R \quad \text{and} \quad Z_3 = Z_4 = C$$

As shown in fig (2) above.

The "Transfer-function" of this network takes the following form:-

$$A_v(s) = \left[(A_{v_0}) \left\{ \frac{\left(1/RC\right)^2}{s^2 + \left(\frac{3 - A_{v_0}}{RC}\right)s + \left(\frac{1}{RC}\right)^2} \right\} \right] \quad - (8)$$

Comparing eq^n (8) with eq^n (5) we obtain,

$$\omega_0 = 1/RC \quad — (9)$$

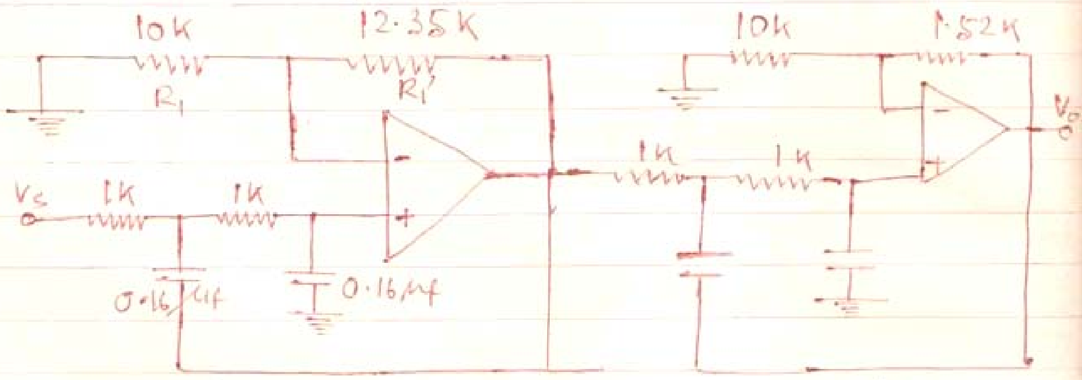
$$\& \quad 2K = (3 - A_{v_0}) \quad \text{i.e.} \quad A_{v_0} = 3 - 2K \quad — (10)$$

Thus even order Butterworth filters may be constructed by cascading circuits in FIG: 2 (i.e. FIG: 3) using identical R & C.

Designing of the active low pass filter of fourth order:

In order to design a fourth order Butterworth filter with a cut-off frequency of 1 KHz, we have the following solution.

We cascade two second order ~~filter~~ circuits of FIG: 2 & obtain the circuits shown in fig 4



4th order (n = 4) Butterworth low pass filter (with f_0 = 1 KHz)

Now, if $R_1 = 10K$ $\quad R_1' = 12.35K$

If $R_2' = 1.52$; $\quad R_2 = 10K$