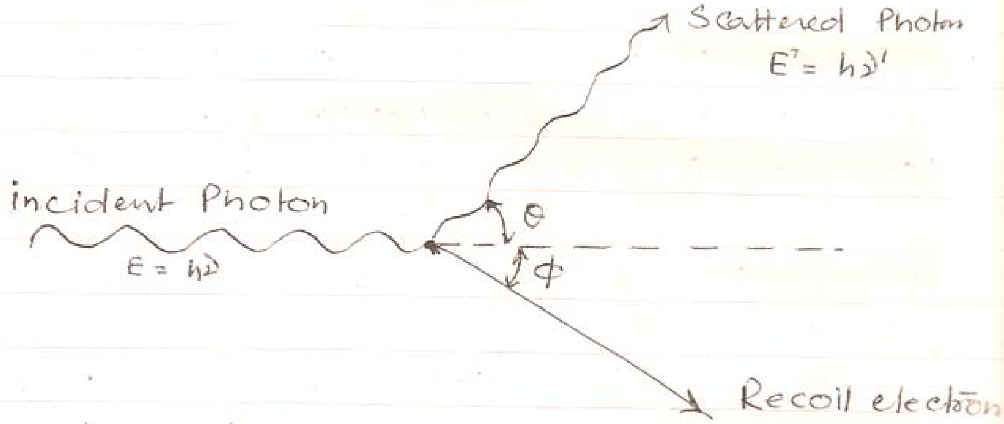
(2) COMPTON EFFECT: When the energy of the γ-ray photon is about 0.5 MeV there is a greter tendency for interaction to take place with individual electrons. When a photon of energy $h\nu$ strikes the perfectly free electron (at rest) the photon with diminished energy '$h\nu'$' is scattered at an angle 'θ' with the direction of incident photon while the electron recoils at an angle 'ϕ' as shown in the fig.



From the laws of conservation of energy and momentum in directions both parallel and perpendicular to the incident photon the change in wave length for the scattered photon is given by the relation:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad \text{—— (8)}$$

where $\Delta\lambda$ is known as Compton Shift & $\frac{h}{m_e c}$ is constant called Compton wavelength (=

Compton Shift in wave length is independent of the incident wave length and material of the scatterer. It is dependent on angle θ. Since $\lambda = \frac{c}{\nu}$

here : $\frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_e c}(1-\cos\theta)$

* or $\left[h\nu' = \frac{h\nu}{1+(1-\cos\theta)\varepsilon_0} \right]$

$$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_e c^2}(1-\cos\theta)$$

$$\frac{\nu}{\nu'} - 1 = \frac{h\nu}{m_e c^2}(1-\cos\theta)$$

$$\frac{\nu}{\nu'} = 1 + \varepsilon(1-\cos\theta)$$

$$\therefore \boxed{h\nu' = \frac{h\nu}{1+(1-\cos\theta)\varepsilon_0}}$$

where $\varepsilon_0 = \frac{h\nu}{m_e c^2}$.

The K.E of the recoil electron is given by

$$\boxed{T = h\nu - h\nu' = h\nu \frac{(1-\cos\theta)\varepsilon_0}{1+(1-\cos\theta)\varepsilon_0}} \quad \text{--- (9)}$$

The K.E of the electron has its maximum value when $\theta = 180^\circ$ and photon is scattered directly backward. The energy is zero for $\theta = 0$ and $\nu = \nu'$

$$\therefore \quad T_{max} = \frac{h\nu}{1+\frac{1}{2}\varepsilon} \quad \text{--- (10)}$$

The angles θ and ϕ can be related as

$$\cot\phi = (1+\varepsilon_0)\tan\frac{\theta}{2} \quad \text{--- (11)}$$

$42621 \times 10^{-12} m)]$ The differential cross-section per electron in the case of linearly polarized incident plane electromagnetic wave is given by Klein and Nishima with the idea of Quantum mechanics as

$$[\sigma_c(\theta)]_{Pol} = \left[\frac{d\sigma_c}{d\Omega}\right]_{Pol} = \frac{1}{4} r_0^2 \left(\frac{\nu'}{\nu}\right)^2 \left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} + 4\cos^2\theta - 2\right)$$

where Θ is the angle between the direction of incident polarization and emergent ray.

for unpolarized incident wave:

$$[\sigma_c(\theta)]_{unpol} = \frac{1}{2} r_0^2 \left(\frac{\nu'}{\nu}\right)^2 \left(\frac{\nu}{\nu'} + \frac{\nu'}{\nu} - \sin^2\theta\right)$$

$$= \frac{1}{2} r_0^2 \cdot \frac{-\varepsilon\cos^3\theta + (\varepsilon^2+\varepsilon+1)(1+\cos^2\theta) - \varepsilon(2\varepsilon+1)}{[1+\varepsilon(1-\cos\theta)]^3}$$ $\boxed{\cos\theta}$

For low energies $\varepsilon \to 0$ the above relation reduces to the Thomson cross-section.

∴ Total cross-section is given by integrating over all angles $(0 < \theta < \pi)$

Or $$\sigma_c = \int \sigma_c(\theta)\, d\Omega = \int_0^{\pi} \sigma_c(\theta)\, 2\pi \sin\theta\, d\theta$$

$$= 2\pi r_0^2 \left\{ \frac{1+\varepsilon}{\varepsilon^2} \left[\frac{2(1+\varepsilon)}{1+2\varepsilon} - \frac{1}{\varepsilon} \log_e(1+2\varepsilon) \right] + \frac{1}{2\varepsilon} \log_e(1+2\varepsilon) - \frac{1+3\varepsilon}{(1+2\varepsilon)^2} \right\} \quad (12)$$

for small value of ε eq^n (12) becomes:

$$\sigma_c \simeq \frac{3}{8} \pi r_0^2 \left(1 - 2\varepsilon + 5.2\varepsilon^2 - 13.2\varepsilon^3 + \cdots\right) \quad (13)$$

Since the scattered energy is smaller than incident energy by a factor $\frac{h\nu'}{h\nu}$. Hence the energy scattering cross-section:

$$\sigma_s = \left(\frac{h\nu'}{h\nu}\right) \sigma_c \quad (14)$$

The probability for the recoil energy to be imparted to the electron in this process is given by the energy absorption cross-section σ_a $(= \sigma_c - \sigma_s)$.

The atomic Compton cross-section is given by

$$_a\sigma_c = Z\sigma_c \quad (15)$$

For the Scattering to be coherent $h\nu = h\nu'$
and for the scattering to be incoherent. $h\nu' < h\nu$
In coherent scattering individual amplitudes for each
case of the atomic electrons are added and in the latter
case the intensities are added.
Thus the atomic differential Compton cross-section is
defined as:

$$d\sigma(\theta) = d\sigma_{coh}(\theta) + d\sigma_{incoh}(\theta) \quad \text{— (16).}$$

Thus the total scattering coefficient per electron
decreases with increasing photon energy and this decrease
is quite slow at low values of energy and for energies
above 0.5 MeV, σ_c is roughly proportional to $(h\nu)^{-1}$.