



Pair Production

③ PAIR PRODUCTION: When γ -ray interacts with matter having energy greater than $2m_0c^2 \approx 1.02\text{MeV}$.
⑫ It creates an electron-positron pair. This pair creation is the process which indicates the transformation of energy into matter.

If an electron be removed from the negative ~~negative~~ energy states by an external field then there creates a hole in that place. Since this hole is formed by the absence of an electron it behaves like a positively charged particle whose mass is equal to the mass of electron & carries positive charge equal to the electronic charge. This antiparticle of electron is named POSITRON (e^+). It has acquired an energy E , momentum P and charge e .

This process can take place only when the photon energy exceeds $2m_0c^2$ which is the sum of the rest energies of the members of the pair.

Applying the Energy Conservation principle:

$$E_\gamma = h\nu = 2m_0c^2 + E^+ + E^- \quad (17)$$

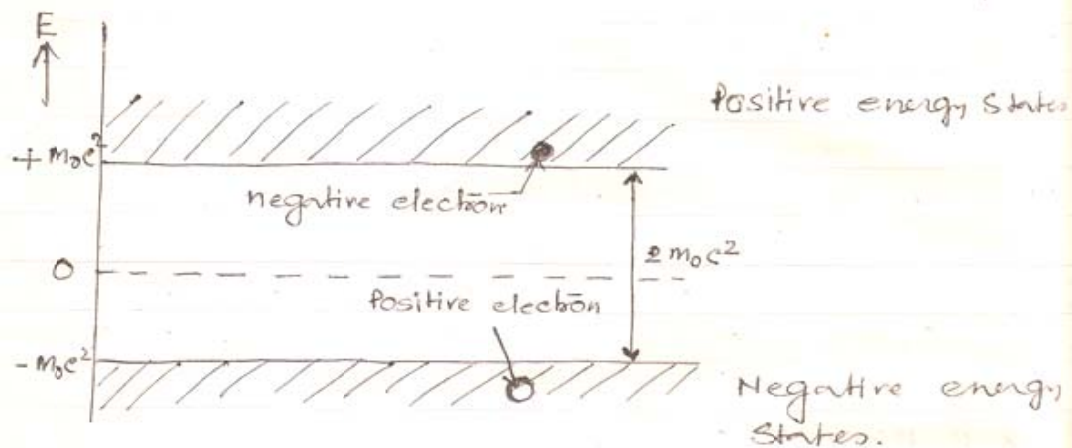


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where E_+ = K.E of positron.
 E_- = K.E of negatōn

In eqⁿ (17) the energy carried by the recoil nuclei is neglected.

The photon energy in excess of $2m_0c^2$ is shared almost equally between the two particles.



Positive and negative energy states of electron according to Dirac's theory.

while positron receives slightly more energy as it is repelled by the nucleus than the negatōn which is attracted. This difference decreases when the photon energy increases.

Dirac's relativistic wave equation gives us energy eigen values for the free electron as

$$E = \pm (p^2c^2 + m_0^2c^4)^{1/2} \quad \text{--- (18)}$$

These values ranges from $-\infty$ to $-m_0c^2$ & then from $+m_0c^2$ to $+\infty$ with a gap of $2m_0c^2$

According to Dirac in the absence of an external field all negative energy states were filled with electrons. The completely filled sea of electrons in the (-)ve region does not contribute to the total energy and momentum of the system.



Pair Production

The electron in the positive energy region cannot make the transition to the (-)ve region. The transition from (-)ve state to (+)ve energy state can take place if a sufficient energy ($\geq 2m_0c^2$) is supplied to the system.

The differential cross-section per nucleus for the creation of a positron of K.E E_+ and a negation of K.E E_- is given by:

$$d\sigma_{pp} = \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 \frac{Z^2 P dE_+}{(h\nu - 2m_0 c^2)} \quad (19)$$

where P is a complicated function of $h\nu$ and Z varies between 0 and about 20. The total pair production cross-section

$$\begin{aligned} \sigma_{pp} &= \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 Z^2 \int_0^{h\nu - 2m_0 c^2} \frac{P dE_+}{h\nu - 2m_0 c^2} \\ &= \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 Z^2 \bar{P} \quad (20) \end{aligned}$$

Integration of (19) for extremely relativistic case gives

$$\sigma_{pp} = \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 Z^2 \left[\frac{28}{9} \log_e \frac{2h\nu}{m_0 c^2} - \frac{218}{27} \right] \quad (21)$$

Applying screening correction for high energy photons (which reduces the effective nuclear charge)

$$\sigma_{pp} = \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 Z^2 \left[\frac{28}{9} \log_e \frac{183}{Z} - \frac{2}{27} \right] \quad (22)$$

when the pair creation occurs in the field of an electron at an energy $> 4m_0 c^2$ then total cross-section per electron: (without screening)

$$e\sigma_{pp} = \alpha \left(\frac{e^2}{4\pi\epsilon_0 m_0 c^2} \right)^2 \left(\frac{28}{9} \log_e \frac{2h\nu}{m_0 c^2} - 11.3 \right) \quad (23)$$

& for the whole atom:

$$\sigma_{ip} = Z e\sigma_{pp}$$



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Hence for an atom of atomic no. Z :

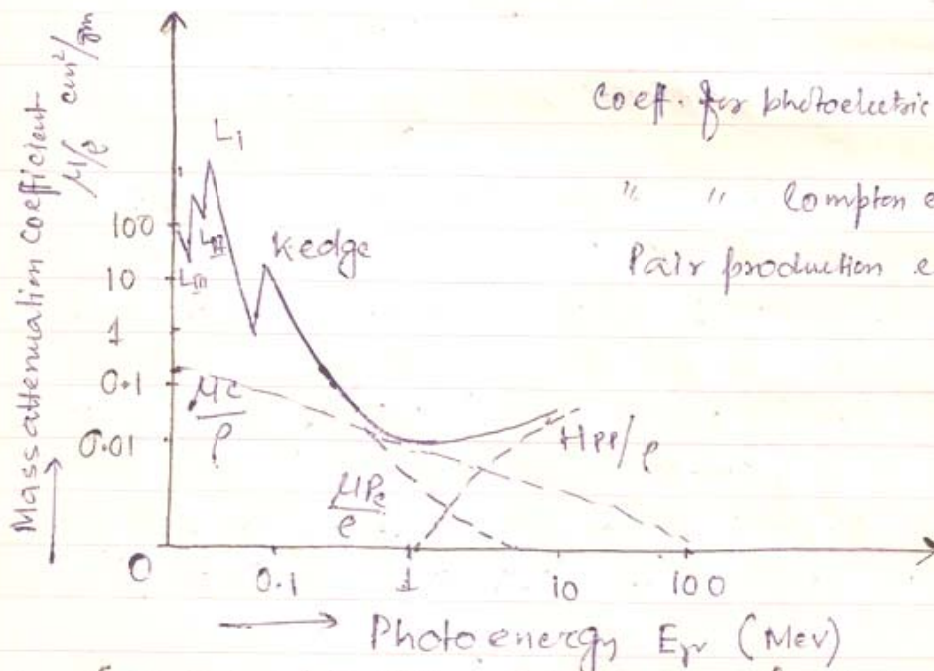
$$\frac{\text{Total cross-section for all } Z \text{ electrons}}{\text{Total cross-section for nuclear pair production}} = \frac{1}{Z} \quad (2)$$

The total linear attenuation coefficient μ

$$\mu = N(\sigma_R + \sigma_{pe} + \sigma_{pp}) + N Z \sigma_e \quad (25)$$

where N = no. of atoms per unit volume of the absorber.

Specification of energy region in which each of the process has a dominant concentration:



Coeff. for photoelectric effect = $\frac{\mu_{pe}}{e}$
 " " Compton effect = $\frac{\mu_c}{e}$
 Pair production effect = $\frac{\mu_{pp}}{e}$

(Total absorption coefficient of Lead)
 Mass attenuation coefficient for γ -rays in lead as a function of energy is shown in the fig. The coeff. for the three important effects shown by dotted curves.

The absorption by photoelectric effect predominates at small energies. The probability of photoelectric $\propto \frac{1}{E^3}$.



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Emission increases as $h\nu \rightarrow E_B$. Above the absorption edge the probability of photoelectric emission varies roughly as $Z^5 E_\nu^{-3.5}$. In the energy region below 0.1 Mev, we get edges. At these edges cross-section shows discontinuous jump. The L_I, L_{II}, L_{III} edges are corresponding to electrons from L-orbit. Photoelectric effect decreases rapidly with increase in energy.

At energies (0.5 Mev to 1 Mev) most of the effect, attenuation is caused by Compton effect.

Absorption by pair production starts at about 1 Mev and increases until at higher energies; the absorption is almost completely by pair production only.

CONCLUSION :

- (1) Photoelectric effect dominates at low E and high Z.
- (2) Compton effect dominates at medium energies E & low Z.
- (3) Pair production effect dominates at high energies E & high Z.