Q1. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer:

Considering $x = \tan^{-1}(1) = x$ $or\ \tan x = 1$ or $\tan x = \tan\left(\frac{\pi}{4}\right)$ $or\ x = \frac{\pi}{4}$ Also let $y = \cos^{-1}\left(-\frac{1}{2}\right)$ $or\ \cos y = -\frac{1}{2}$ or $\cos y = -\cos\frac{\pi}{3}$ or $\cos y = \cos\left(\pi - \frac{\pi}{3}\right)$ or $\cos y = \cos\frac{2\pi}{3}$ $or\ y = \frac{2\pi}{3}$	**Therefore** $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$ $= x + y$ $= \frac{\pi}{4} + \frac{2\pi}{3}$ $= \frac{11\pi}{12}$ (**Answer**)

Q2. Write the value of $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.

Answer: We know that $2\tan^{-1}\theta = \tan^{-1}\frac{2\theta}{1-\theta^2}$

Hence $2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} = \tan^{-1}\frac{\frac{2}{5}}{\frac{24}{25}} = \tan^{-1}\frac{5}{12}$

$\therefore\ \tan\left(2\tan^{-1}\frac{1}{5}\right) = \tan\left(\tan^{-1}\frac{5}{12}\right) = \frac{5}{12}$

Q3. Find the value of 'a' if $\begin{bmatrix} a-\mathrm{b} & 2a+\mathrm{c} \\ 2a-\mathrm{b} & 3\mathrm{c}+\mathrm{d} \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

Answer: $\begin{bmatrix} a-\mathrm{b} & 2a+\mathrm{c} \\ 2a-\mathrm{b} & 3\mathrm{c}+\mathrm{d} \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

From the left hand and right hand matrix equating corresponding elements, we get

$a-\mathrm{b}=-1 \rightarrow (1)$

$2a+\mathrm{c}=5 \rightarrow (2)$

$2a-\mathrm{b}=0 \rightarrow (3)$

also $3\mathrm{c}+\mathrm{d}=13 \rightarrow (4)$

To find the value of 'a' from equation (3) b = $2a$, putting in equation(1) we get – $a=-1$ or $a=1$

Q4. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x.

Answer: $(x+1)(x+2)-(x-1)(x-3)=12-(-1)$

$or\ x^2+3x+2-x^2+4x-3=13$

$or\ 7x-1=13$

$or\ 7x=14$

$or\ x=2$