Definition

Simple Harmonic Motion (SHM)

A Motion which repeats itself over and over again after a regularly reoccurring time interval along a fixed path is known as Simple Harmonic Motion. It is found that for a particle executing S.H.M the acceleration is proportional to displacement & is always directed towards a fixed point in its path known as mean position.

As it goes towards A, displacement increases acceleration also increases, but the direction of the acceleration being along AO the particle moves against the acceleration consequently the velocity decreases. When velocity becomes zero the particle comes to rest at A. At A , u=0 , acceleration is maximum (because the displacement is maximum). The particle starts moving in the direction of acceleration and velocity increases. The displacement decreases and hence the acceleration decreases. When it comes to 'O', displacement is zero, acceleration is zero but velocity is maximum. The particle continues to move along the direction of velocity i.e. OB, the displacement increases acceleration increases but being along BO velocity decreases, till velocity becomes zero at B. The particle continues to move in this way.

Characteristics of S.H.M

(1) It is to & fro motion

(2) It repeats itself again and again

(3) It is periodic motion

(4) Acceleration α – displacement

The negative sign indicates that the acceleration vector is opposite in direction to the displacement vector.

Displacement: The displacement at any instant of time t is the shortest distance of the Particle at that instant from the mean position.

It is a vector and direction is always away from the mean position represented by x or y.

Amplitude (a) : The maximum displacement on either side of the mean position

is known as amplitude.

Time period (T) : The time taken to make one complete oscillation is known as time period.

Mathematical representation of S.H.M:

A S.H.M can be mathematically represented by a uniform circular motion of a vector along a circle such that the foot of the perpendicular dropped from the tip of the radius vector on a diameter executes the S.H.M. This imaginary circle is known as the circle of reference.

Displacement Equation: The relation from which we can find the displacement of the particle in SHM at any instant of time't' is known as displacement equation.

Let us consider a S.H.M being executed along Y axis.

Given:

a = amplitude of the S.H.M

T = Time period of the S.H.M

$\omega = \frac{2\pi}{T} =$ Angular frequency of the S.H.M i.e. the angular velocity of the radius vector of the uniform circular motion which mathematically represent the given Simple Harmonic Motion.

Let us represent the given S.H.M by a uniform circular motion of radius '*a*' equal

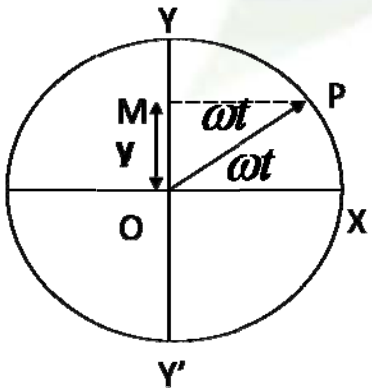
to the amplitude of the given S.H.M. Starting from OX let the radius vector come to OP at any instant of time t.

$\angle XOP = \omega t$

From the tip (P) of the radius vector, drop a perpendicular on diameter YOY' the foot of the

Perpendicular M represents the position of the particle Executing S.H.M.

M is the position of the particle at any instant t, OM=y = displacement at an instant t



$\Delta OPM, Sin\omega t = \frac{y}{a}$

$y = aSin\omega t \longrightarrow$ (1)

Equation (1) is known as displacement equation of a S.H.M. If the oscillation is along X – axis then

x = $aSin\omega t$

Graphical Representation: Let T = time period

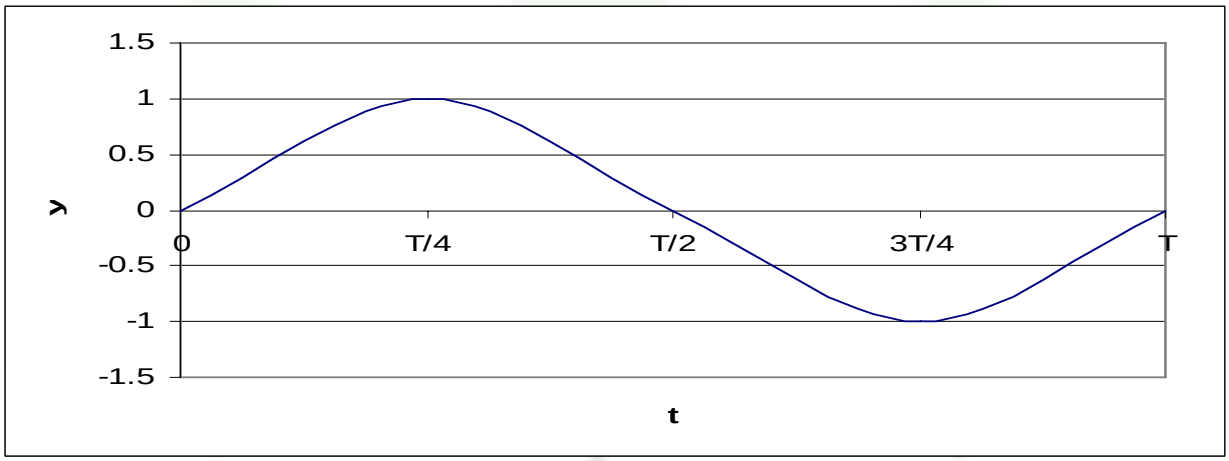
$t = 0, \omega t = 0, Sin\omega t = Sin0 = 0, y = 0$

$t = \frac{T}{4}, \omega t = \frac{\pi}{2}, Sin\omega t = Sin\frac{\pi}{2} = 1, y = a$

$t = \frac{3T}{2}, \omega t = \pi, Sin\omega t = Sin\pi = 0, y = a.0 = 0$

$t = \frac{3T}{4}, \omega t = \frac{3\pi}{2}, Sin\omega t = Sin\frac{3\pi}{2} = -1, y = a.(-1) = -a$

$t = T, \omega t = 2\pi, Sin\omega t = Sin2\pi = 0, y = 0$



The graphical or pictorial representation of the displacement equation is a wave, in other words displacement equation is the mathematical representation of the wave.