Capacitance of a conductor: To charge a body it is to be connected to a source of potential. It is found that higher is the potential applied greater is the amount of charge stored in that body. Moreover when same potential is applied on different bodies' different amount of charge is stored in them.

$$Q \propto V \; or \; Q = C.V \rightarrow (1)$$

Where C is constant of proportionality and is known as the capacitance of the body and is different for different bodies.

Capacitance of a body depend on

(i) The geometry of the body (size & shape)

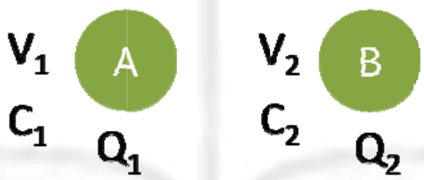
(ii) The nature of surrounding medium

If V=1 volt from equation (1) Q=C

Hence capacitance of a body can be defined as the amount of charge stored in the body when 1 volt potential is applied on it.

Unit is Farad. Coulomb =Volt x farad

Distribution of charge when two bodies of different potentials are connected



Let us consider two bodies A & B at different potentials.

V_1 & V_2 =Potentials of A & B respectively

C_1 & C_2 =Capacitance of A & B respectively

Q_1 & Q_2 =Charge stored in A & B respectively

Let the two bodies be connected by a conducting wire charge flows from the body at higher potential to the body at lower potential till both attain the same potential.

Let V = common potential

Q_1' and Q_2' =the new charge in A and B respectively.

$Q_1 = C_1V_1$

$Q_2 = C_2V_2$

$Q_1' = C_1V$

$Q_2' = C_2V$

Since flow of charge is conserved

$Q_1 + Q_2 = Q_1' + Q_2'$

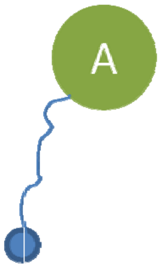
$C_1V_1 + C_2V_2 = C_1V + C_2V$

$V = (C_1V_1 + C_2V_2)/(C_1 + C_2)$

$$Q_1' = C_1 \frac{(C_1V_1 + C_2V_2)}{(C_1 + C_2)} \rightarrow (2)$$

$$Q_2' = C_2 \frac{(C_1V_1 + C_2V_2)}{(C_1 + C_2)} \rightarrow (3)$$

Energy Stored In Charged Body:



Source of Potential V

Given C = Capacitance of the body.

V= Potential of the source

Q= the maximum charge stored in the body when it's potential rises to V

Let during charging q be the charge stored in the body at any instant t and v be the potential of the body at that instant.

$$q = cv$$

$$v = \frac{q}{c} \rightarrow (1)$$

The charge q which is already in the body will repel the like charge dQ as it comes from the source hence work is to be done in storing the charge dQ

$$\text{Work done dw} = \text{v.dQ} = \frac{q}{C}\text{dQ} \rightarrow (2)$$

Hence total work done in storing the charge Q can be obtained by integrating equation(2)

$$W = \int dw = \int_0^Q \frac{q}{C} dQ = \frac{1}{2}\frac{Q^2}{C} \rightarrow (3)$$

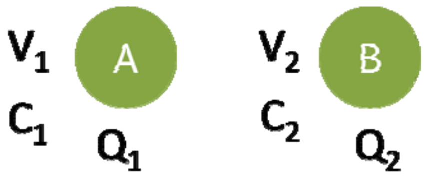
This work done is stored in the charged body in the form of electrostatic potential energy.

$$\therefore E = \frac{1}{2}\frac{Q^2}{C} \rightarrow (4)$$

$$Q = CV$$

$$E = \frac{1}{2}\frac{C^2V^2}{C} = \frac{1}{2}CV^2 \rightarrow (5)$$

Loss of energy when two bodies at different potentials are connected:



Let us consider two bodies A & B at different potentials.

V_1 & V_2 =Potentials of A & B respectively

C_1 & C_2 =Capacitance of A & B respectively

Q_1 & Q_2 =Charge stored in A & B respectively

E_1 & E_2 = Energy stored in A & B respectively

$$Q_1 = C_1V_1$$
$$Q_2 = C_2V_2$$
$$E_1 = \frac{1}{2}C_1V_1^2$$
$$E_2 = \frac{1}{2}C_2V_2^2$$

Total energy before they are connected $E = E_1 + E_2$

$$E = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \rightarrow (1)$$

When two bodies are connected by a conducting wire charge flows from the body at higher potential to the body at lower potential till both attain the same potential.

Let V be the common potential

$Q_1^{'}$ & $Q_2^{'}$ = New charge stored in A & B respectively.

$Q_1^{'} = C_1V$

$Q_2^{'} = C_2V$

Since the flow of charge is conserved

$Q_1 + Q_2 = Q_1^{'} + Q_2^{'}$

$C_1V_1 + C_2V_2 = C_1V + C_2V$

$$V = \frac{C_1V + C_2V}{C_1 + C_2} \rightarrow (2)$$

$$\therefore E_1^{'} = \frac{1}{2}C_1V^2 \ \& \ E_2^{'} = \frac{1}{2}C_2V^2$$

The energy after connection: $E^{'} = E_1^{'} + E_2^{'}$

$$\text{or } E^{'} = \frac{1}{2}V^2(C_1 + C_2)$$

$$E^{'} = \frac{1}{2}(C_1 + C_2)\left[\frac{(C_1V + C_2V)^2}{(C_1 + C_2)^2}\right]$$

$$E^{'} = \frac{1}{2}\frac{(C_1V + C_2V)^2}{(C_1 + C_2)} \rightarrow (3)$$

Loss of energy $\Delta E = E - E^{'}$

$$\Delta E = \frac{1}{2}\left[C_1V_1^2 + C_2V_2^2 - \frac{(C_1V + C_2V)^2}{(C_1 + C_2)}\right]$$

$$\Delta E = \frac{C_1C_2}{2(C_1 + C_2)}\left[V_1^2 + V_2^2 - 2V_1V_2\right]$$

$$\Delta E = \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2 \rightarrow (4)$$

Discussions:

(i)If the two bodies at same potential are connected $V_1=V_2$ then $\Delta E=0$ hence no energy is lost

(ii) The loss of energy takes place in the form of heat energy light energy through sparking