Statement: For a small amount of non viscous liquid flowing from one point to other without any friction in stream line motion, the sum total of three energies i.e. gravitational potential energy per unit mass, kinetic energy per unit mass and pressure energy per unit mass remains constant at every point during its motion.

Explanation:

(1) Since a liquid possesses inertia hence it possesses gravitational P.E.

m = mass of the liquid, h = height above the reference level.

Gr. P.E = mgh

Gr. P.E per unit mass = gh

(2) When the liquid flows due to its motion it possesses K.E

u = velocity of flow

K.E = ½ mu^2

K.E per unit mass = ½ u^2

(3) Since a liquid can exert hydrostatic pressure it is capable of doing work and possesses an energy known as pressure energy.

It can be shown that the pressure energy per unit mass = P/ρ

Where P = Pressure exerted by the liquid

ρ = density of the liquid

$$\left\{ \begin{array}{l} P = \frac{F}{A} = \frac{Ah\rho g}{A} \\ \frac{P}{\rho} = gh \end{array} \right.$$

According to Bernoulli's theorem

$$gh + \frac{1}{2}u^2 + \frac{P}{\rho} = \text{constant}$$

Proof: (1) We first show that the Gr. P.E & pressure energy are mutually convertible.

Let us consider a liquid taken in a container.

h = height of the free surface of the liquid above the bottom of the container.

Consider a small amount of liquid of mass m at the free surface

Gr. P.E = mgh, Gr. P.E per unit mass = gh

Pressure exerted at the bottom P =ρgh

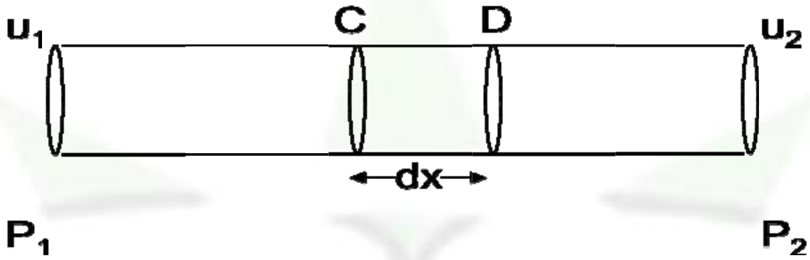
Pressure energy per unit mass $= P/\rho = \rho gh/\rho = gh$

From equation (1) and (2) we see that

Gr. P.E per unit mass = Pressure energy per unit mass

(2) We now show that K.E & pressure energy are mutually convertible from one form to the other

Let us consider a stream line flow of the liquid through a tube of uniform cross section.



Given: α = cross-sectional area of the tube

P_1 and P_2 = Pressure at the inlet & outlet of the tube

u1 and u2 = The velocity of flow of the liquid at the inlet and outlet of the tube

ρ = density of the liquid.

Let us consider a an element CD of the liquid tube perpendicular to the axis of length dx

Let P = Pressure at the point C.

$\frac{dp}{dx}$ = Pressure gradient at C { Rate of change of pressure with distance, when distance changes by 1 pressure changes by $\frac{dp}{dx}$

The pressure at D $= P + \frac{dp}{dx}dx$

The pressure on liquid element CD in the direction of flow = Pressure at C – Pressure at D

$$= P - \left(P + \frac{dp}{dx}dx\right) = -\frac{dp}{dx}dx$$

The negative sign indicates that the pressure decreases as the distance increases.

The force on the liquid element due to this pressure difference $= -\frac{dp}{dx}dx.\alpha$

Mass of the liquid element $= (\alpha dx)\rho$

The force due to pressure difference acting on the liquid element produces acceleration $\frac{du}{dt}$

Using Newton's Second law

$$(\alpha dx\rho)\frac{du}{dt} = -\frac{dp}{dx}dx\alpha$$

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{dp}{dx}$$

$$\frac{dx}{dt}\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$$

$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$$

$$udu = -\frac{dp}{\rho}$$

Integrating both sides

$$\int_{u_1}^{u_2} udu = -\int_{P_1}^{P_2}\frac{dp}{\rho}$$

$$\frac{u_1^2}{u_2^2} = -\frac{1}{\rho}\left(P_2 - P_1\right)$$

$$\frac{1}{2}u_1^2 + \frac{P_1}{\rho} = \frac{1}{2}u_2^2 + \frac{P_2}{\rho}$$

(K.E per unit mass + Pressure energy per unit mass) $_{inlet}$ = (K.E per unit mass + Pressure energy per unit mass) $_{outlet}$

Thus K.E and Pressure energy are mutually convertible from one form to the other. Thus the three forms of energy are mutually convertible from one form to the other hence Bernoulli's theorem is proved

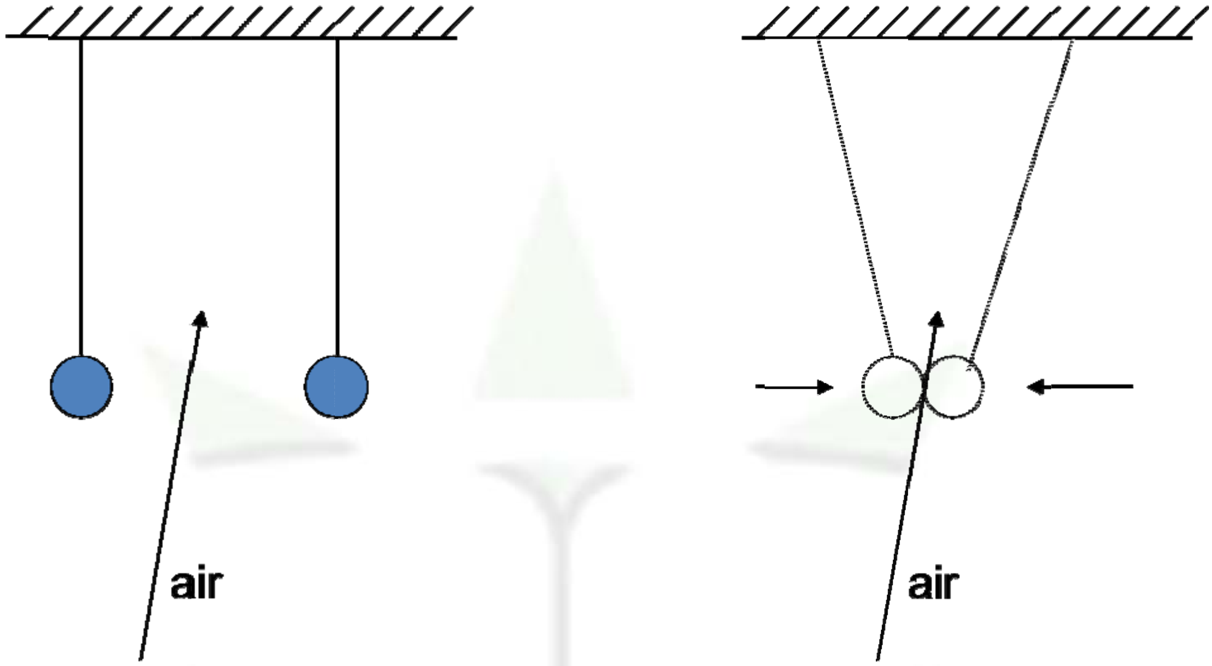
Discussion:

$$\frac{1}{2}u_1^2 + \frac{P_1}{\rho} = \frac{1}{2}u_2^2 + \frac{P_2}{\rho}$$

If $u_1 > u_2$ then $P_1 < P_2$

The velocity of flow is greater in the region of lower pressure and vice versa.

(A) When air is blown between two suspended objects, velocity increases pressure drops in that region, air flows for surrounding (higher pressure) and objects comes closure

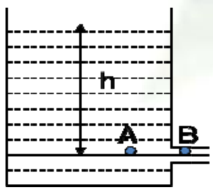


(B) A person standing on the platform close to running train gets attracted by the train

Application of Bernoulli's theorem:

(1) **Velocity of efflux**:

The velocity with which the liquid comes out through the hole near the bottom is known as velocity of efflux.



Let the axis of the hole be taken as the reference level. When the liquid comes out through the hole the level of the liquid in the container falls so slowly liquid inside the container can be assumed to be at rest.

Given:

P_o = atmospheric pressure

h = height of the free surface of liquid above the reference level.

V_e = velocity of efflux

ρ = density of the liquid

Let us consider two points on the axis of the whole A being inside container and B just outside the hole.

	A	B
Gr. P.E per unit mass of the liquid at	0	0
K.E of the liquid per unit mass at	0	$\frac{1}{2}V_e^2$
Pressure energy per unit mass at	$\left(\frac{P_o + h\rho g}{\rho}\right)$	$\frac{P_o}{\rho}$

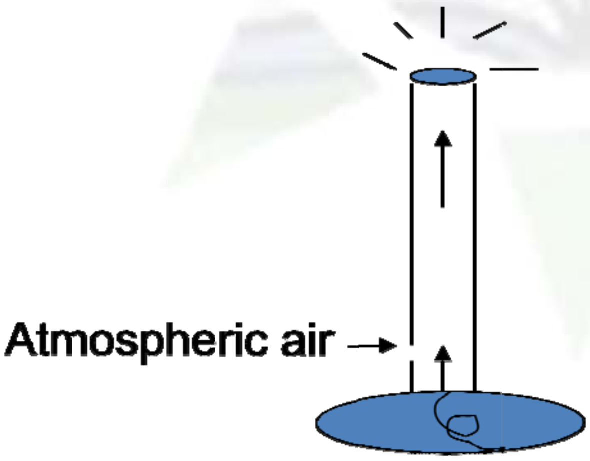
Applying Bernoulli's theorem

$$0 + 0 + \frac{P_o}{\rho} + hg = 0 + \frac{1}{2}V_e^2 + \frac{P_o}{\rho}$$

$$V_e = \sqrt{2hg}$$

(2) **Oxidizing flame in Bunsen burner**

To get oxidizing flame in Bunsen burner the air hole near the bottom of the burner tube is opened where by oxygen in the atmospheric air enters into the tube through the hole and mixes with the burning gas to give the oxidizing flame.



The velocity of the burning gas flowing through the burner being very high the pressure of the gas inside the tube is lesser than that of outside the tube. Hence air from outside rushes into the tube.