13. Let f(x) be a polynomial of degree four having extreme values at x = 1 and x =2. If $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$ Then f (2) is equal to:

(1) – 8 (2) – 4 (3) 0 (4) 4

Answer:

Let the 4th degree polynomial be

$f(x)=ax^4+bx^3+cx^2+dx+k$

Given $\lim_{x\to 0}\left[1+\frac{f(x)}{x^2}\right]=3$

$or\ \lim_{x\to 0}\left(1+\frac{ax^4+bx^3+cx^2+dx+k}{x^2}\right)=3$

$or\ \lim_{x\to 0}\left(\frac{ax^4+bx^3+cx^2+dx+k}{x^2}\right)=2$

$or\ c=2.d=0,k=0$

$hence\ f(x)=2x^2+bx^3+ax^4$

$Differentiating\ f'(x)=4x+3bx^2+4ax^3$

$f'(1)=4+3b+4a=0\rightarrow(1)$

$f'(2)=8+12b+32a=0$

$or\ 2+3b+8a=0\rightarrow(2)$

$from\ (1)\ and\ (2)\ we\ get\ 2-4a-4+8a=0$

$or\ 4a=2,a=\frac{1}{2}\ also\ 4+3b+2=0\ ,b=-2$

$Hence\ f(x)=2x^2-2x^3+\frac{1}{2}x^4\ or\ f(2)=8-16+8=0$

Correct option is (3) 0