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17. Let y(x) be the solution of the differential equation $(xlogx)\frac{dy}{dx} + y = 2xlogx, (x \geq 1)$ Then y(e) is equal to

(1) e (2) 0 (3) 2 (4) 2e

Answer: The given equation can be written in the form of standard differential equation as

$$\frac{dy}{dx} + \frac{y}{xlogx} = 2 \rightarrow (1)$$

$$Integrating\ factor\ (I.F) = e^{\int \frac{1}{xlogx}dx} = e^{log(logx)} = logx$$

$$\left[\int \frac{1}{xlogx}dx = \int \frac{du}{u}, where\ u = logx, so\ du = \frac{1}{x}dx, hence\ logu = \log(logx)\right]$$

Multiplying both sides of equation (1) by integrating factor and integrating with respect to x

$$\int \left((logx)\frac{dy}{dx} + \frac{y}{x}\right)dx = \int 2logxdx$$

$$or\ ylogx = 2(xlogx - x) + c \rightarrow (2)$$

For limiting value at x=1 from given equation we find y=0 , therefore putting (1,0) in equation(2) we get 0=-2+c or c=2, hence equation becomes

$$ylogx = 2(xlogx - x) + 2$$

$$thefore\ at\ x = e\ , yloge = 2(eloge - e) + 2, y = 2$$

Correct option is (3) 2