SelfStudy.in

15. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ *and* $\frac{V_0}{4}$ and have radius R_1, R_2, R_3 and R_4 respectively. Then

(1) $R_1 = 0$ and $R_2 > (R_4 - R_3)$ (2) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$
(3) $R_1 = 0$ and $R_2 < (R_4 - R_3)$ (4) $2R < R_4$

Answer:

We know that potential on the surface of the solid sphere is given by

$$V_0 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \rightarrow (1)\ at\ r = R$$

when r > R then potential at r from center

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \rightarrow (2)$$

also when r < R we have

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R^3}(3R^2 - r^2) \rightarrow (3)$$

when $r = R_1$ *we have Potential*

$$\frac{3V_0}{2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R_1}$$

$$or\ \frac{3V_0}{2} = \frac{RV_0}{R_1}$$

$$or\ R_1 = \frac{2}{3}R$$

when $r = R_2$ *we have*

$$\frac{5V_0}{4} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R^3}(3R^2 - R_2^{\ 2})$$

$$or\ \frac{5V_0}{4} = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R^3}(3R^2 - R_2^{\ 2})$$

$$or\ \frac{5V_0}{4} = V_0 \times \frac{1}{2R^2}(3R^2 - R_2^{\ 2})$$

$$or\ \frac{5R^2}{2} - 3R^2 = -R_2^{\ 2}$$

$$or\ \frac{R^2}{2} = R_2^{\ 2}$$

$$or\ R_2 = \frac{R}{\sqrt{2}}$$

$$when\ r = R_3\ then\ \frac{1}{4\pi\epsilon_0}\frac{Q}{R_3} = \frac{3V_0}{4}$$

$$or\ \frac{RV_0}{R_3} = \frac{3V_0}{4}$$

$$or\ R_3 = \frac{4}{3}R$$

$$when\ r = R_4\ then\ \frac{1}{4\pi\epsilon_0}\frac{Q}{R_4} = \frac{V_0}{4}$$

$or\ \frac{RV_0}{R_4} = \frac{V_0}{4}\ or\ R_4 = 4R$, $\boldsymbol{R_4 > 2R}$

Now $R_4 - R_3 = 4R - \frac{4}{3}R = \frac{8R}{3}$

therefore $\boldsymbol{R_4 - R_3 > R_2}$

also $R_2 - R_1 = \frac{R}{\sqrt{2}} - \frac{2}{3}R$ *therfore*

$\boldsymbol{R_2 - R_1 < R_4 - R_3}$

Correct answer is options (3) and option (4)