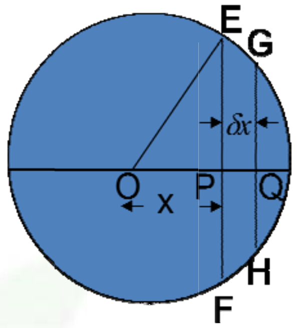
**M.I of a solid sphere**

(A) About a diameter of the sphere

Let the diameter XX' be the chosen axis. Given:
M = mass of the sphere

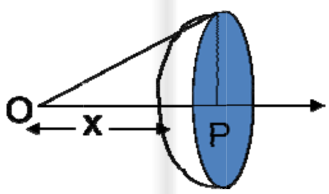
R = Radius of the sphere




Volume $= \frac{4}{3}\pi R^3$

Mass per unit volume $= \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$

Let us cut the sphere by drawing two imaginary planes EF and GH perpendicular to the diameter XX$'$ at a distance x and x + δx from the center O. The elementary slice cut in this way appears like a thin disc as shown below




The radius of the elementary disc = EF

Thickness =PQ = δx

From ΔEOP: $EP^2 = (R^2 - x^2)$

Volume of this elementary disc = $\pi . EP^2 . PQ = \pi(R^2 - x^2)\delta x$

Mass of this elementary circular disc = $\frac{3M}{4\pi R^3}\pi(R^2 - x^2)\delta x$

Since the given axis passes through the center of the elementary disc & perpendicular to the plane of elementary disc hence M.I of this elementary disc about the given axis

$$= \frac{3M}{4\pi R^3}\pi(R^2 - x^2)\delta x \frac{EP^2}{2}$$

$$= \frac{3M}{4\pi R^3}\pi(R^2 - x^2)\delta x \frac{(R^2 - x^2)}{2}$$

$$= \frac{3M}{4\pi R^3}\frac{\pi(R^2 - x^2)^2}{2}\delta x$$

$$= \frac{3M}{8R^3}\left[R^4 + x^4 - 2R^2x^2\right]\delta x$$

Since the given sphere can be assumed to be made up of many such elementary disc, M.I of the sphere can be obtained by integrating equation (1) between the limits x=0 to x=R and multiplying by 2

$$I = 2\int_0^R \frac{3M}{8R^3}\left[R^4 + x^4 - 2R^2x^2\right]\delta x$$

$$= \frac{6M}{8R^3}\left[\int_0^R R^4\delta x + \int_0^R x^4\delta x + \int_0^R 2R^2x^2\delta x\right]$$

$$= \frac{3M}{4R^3}\left[R^4\{x\}_0^R + \left\{\frac{x^5}{5}\right\}_0^R - \frac{2R^2}{1}\left\{\frac{x^3}{3}\right\}_0^R\right]$$

$$= \frac{3M}{4R^3}\left[R^5 + \frac{R^5}{5} - \frac{2}{3}R^5\right]$$

$$= \frac{2MR^2}{5}$$

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M.I of a hollow shell about its diameter $= \frac{2MR^2}{3}$