Magnetic Field At The Centre Of A Circular Coil Carrying Current

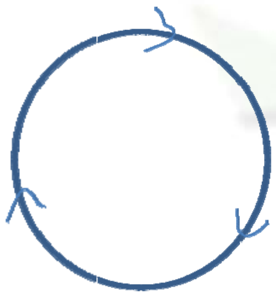
We have a circular coil of many turns a current flow through the coil. We have to find the magnetic field at the centre of the coil due to the current through the coil.

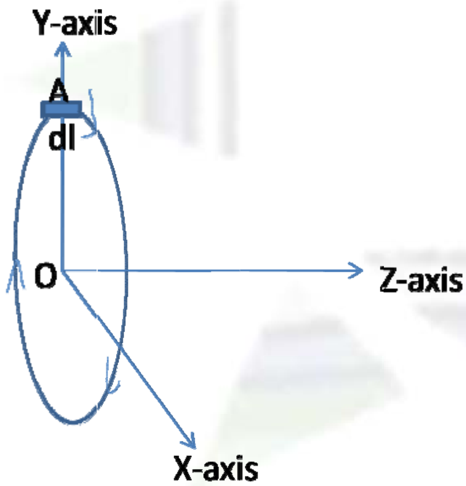
Given

i = current flowing through the coil

a =radius of the coil

n = number of turns in the coil





Let us choose a right handed system of mutually perpendicular axes X,Y and Z as follows. The plane of the coil is represented by Y-Z plane and X-axis is perpendicular to the plane of the coil at O.

Let us consider an element of the coil at A of length dl. The axis of the element is along Z-axis i.e. dl is along Z-axis using Biot's law the magnetic field at O due to the current element at A

$$\mathrm{db} = \frac{\mu_0}{4\pi}\frac{i}{r^2}\left(\overrightarrow{dl} \times \hat{r}\right) \rightarrow (1)$$

Where $\mathrm{r} = \mathrm{a} = \text{radius of the coil}$

$\hat{\mathrm{r}}$ = a unit vector along Y axis

Taking the magnitude of equation(1)

$$\left|\overrightarrow{\mathrm{db}}\right| = db = \frac{\mu_0}{4\pi}\frac{i}{a^2} dl \times 1 \times \sin 90^0 = \frac{\mu_0}{4\pi}\frac{idl}{a^2} \rightarrow (2)$$

Applying the right handed curl rule for vector product the direction of the magnetic field is along X axis in the figure. If we consider the element at different points along the coil it is found that for every position of the element the direction of the magnetic field at the centre O along X-axis.

Since the direction of the magnetic field at O is same for every position of the element hence vector addition reduces to scalar addition and resultant magnetic field at O can be obtained by integrating equation(2)

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{idl}{a^2} = \frac{\mu_0}{4\pi}\frac{i}{a^2}\int dl = \frac{\mu_0}{4\pi}\frac{i}{a^2}(2\pi a \times n) = \frac{\mu_0}{2}\frac{ni}{a} \rightarrow (3)$$

$$\int dl = \text{the total length of the coil} = (2\pi a)n$$

Equation (3) gives the magnitude of the magnetic field at the centre of the circular coil carrying current.

The direction of the magnetic field at the centre which is found to be along X axis can be stated as follows : ”Curl the finger of right hand in the direction of current in the coil the thumb gives the direction of the magnetic field”