**2025**

(v) If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then $A^2$ is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

**Answer:**

Given the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,

we need to find $A^2$.

Calculate $A^2 = A \times A$:

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (0 \times 0 + 1 \times 1) & (0 \times 1 + 1 \times 0) \\ (1 \times 0 + 0 \times 1) & (1 \times 1 + 0 \times 0) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $A^2$ equals the identity matrix.

**Answer (c)**

---

(ii) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -5 & 1 \\ 7 & -4 \end{bmatrix}$ **[4]**

Find :

(a) $A + C$

(b) $B(A + C)$

(c) $5B$

(d) $B(A + C) - 5B$

## Answer:

Let's solve each part step-by-step.

Given matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -5 & 1 \\ 7 & -4 \end{bmatrix}$$

---

**(a)** $A + C$

$$A + C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -5 & 1 \\ 7 & -4 \end{bmatrix} = \begin{bmatrix} 1 + (-5) & 2 + 1 \\ 3 + 7 & 4 + (-4) \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 10 & 0 \end{bmatrix}$$

---

**(b)** $B(A + C)$

First, use $A + C$ from part (a):

$$B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}, \quad A + C = \begin{bmatrix} -4 & 3 \\ 10 & 0 \end{bmatrix}$$

Multiply $B \times (A + C)$:

$$B(A + C) = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} -4 & 3 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} (2 \times -4 + 1 \times 10) & (2 \times 3 + 1 \times 0) \\ (4 \times -4 + 2 \times 10) & (4 \times 3 + 2 \times 0) \end{bmatrix} = \begin{bmatrix} (-8 + 10) & (6 + 0) \\ (-16 + 20) & (12 + 0) \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$$

**(c)** $5B$

Multiply each element of $B$ by 5:

$$5B = 5 \times \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 20 & 10 \end{bmatrix}$$

---

**(d)** $B(A + C) - 5B$

Using the results from (b) and (c):

$$B(A + C) = \begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}, \quad 5B = \begin{bmatrix} 10 & 5 \\ 20 & 10 \end{bmatrix}$$

Subtract:

$$B(A + C) - 5B = \begin{bmatrix} 2 - 10 & 6 - 5 \\ 4 - 20 & 12 - 10 \end{bmatrix} = \begin{bmatrix} -8 & 1 \\ -16 & 2 \end{bmatrix}$$

---

## 2024

(iv) If matrix $A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$, then the value of $x$ is:

(a) 2

(b) 4

(c) 8

(d) 10

**Answer:**

Given:

$$A = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

and

$$A^2 = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$$

**Step 1: Calculate $A^2 = A \times A$**

$$A^2 = \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} (2 \times 2 + 2 \times 0) & (2 \times 2 + 2 \times 2) \\ (0 \times 2 + 2 \times 0) & (0 \times 2 + 2 \times 2) \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix}$$

**Step 2: Compare with given matrix for $A^2$:**

$$\begin{bmatrix} 4 & 8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 4 \end{bmatrix}$$

So,

$$x = 8$$

Answer: (c) 8

**Question 1**

Choose the correct answers to the questions from the given options. [15]

(Do not copy the questions, write the correct answers only.)

(i) If $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, the value of $x$ and $y$ respectively are:

(a) 1, –2

(b) –2, 1

(c) 1, 2

(d) –2, –1

**Answer:**

Given:

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$$

This gives the system of equations:

1. $2x + 0 \cdot y = 2$
2. $0 \cdot x + 4y = -8$

---

Solve each:

1. $2x = 2 \implies x = 1$
2. $4y = -8 \implies y = -2$

---

## Answer:

$x = 1, \quad y = -2$

This matches option **(a)**.

**2023**

**Question 2**

(i) $A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}$ *and* $C = \begin{bmatrix} 4 & 0 \\ x & 1 \end{bmatrix}$ [4]

Find the values of $x$ and $y$, if **AB = C**.

**Answer:**

We are given:

$$A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 0 \\ t & 1 \end{bmatrix}$$

We need to find the values of $x$ and $y$ **such that** $AB = C$.

---

### Step 1: Multiply $A \times B$

$$AB = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} (x \cdot 4 + 0 \cdot y) & (x \cdot 0 + 0 \cdot 1) \\ (1 \cdot 4 + 1 \cdot y) & (1 \cdot 0 + 1 \cdot 1) \end{bmatrix} = \begin{bmatrix} 4x & 0 \\ 4 + y & 1 \end{bmatrix}$$

---

### Step 2: Compare $AB$ with $C$

$$AB = \begin{bmatrix} 4x & 0 \\ 4 + y & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 0 \\ t & 1 \end{bmatrix}$$

Match corresponding elements:

- $4x = 4 \Rightarrow x = 1$
- $4 + y = t$

From $C$, we see that $t = x$, and we just found $x = 1$, so $t = 1$

So:

$$4 + y = 1 \Rightarrow y = 1 - 4 = -3$$

---

### ✅ Final Answer:

$$x = 1, \quad y = -3$$

**Question 4**

(i) $If\ A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}\ and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$ [3]

$Find\ A(B + C) - 14I$

**Answer:**

We are given:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix},\ C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix},\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We are to compute:

$$A(B + C) - 14I$$

---

**Step 1: Compute $B + C$**

$$B + C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$$

---

**Step 2: Compute $A(B + C)$**

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix},\ B + C = \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$$

Multiply:

$$A(B + C) = \begin{bmatrix} (1 \times 5 + 3 \times 3) & (1 \times 3 + 3 \times 9) \\ (2 \times 5 + 4 \times 3) & (2 \times 3 + 4 \times 9) \end{bmatrix} = \begin{bmatrix} 5 + 9 & 3 + 27 \\ 10 + 12 & 6 + 36 \end{bmatrix} = \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix}$$

---

**Step 3: Compute $14I$**

$$14I = 14 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

---

**Step 4: Compute $A(B + C) - 14I$**

$$\begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$$

---

✅ **Final Answer:**

$$\boxed{\begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}}$$

2022

3. **The product $AB$ of two matrices $A$ and $B$ is possible if** **[1]**

(a) $A$ and $B$ have the same number of rows.

(b) the number of columns of $A$ is equal to the number of rows of $B$.

(c) the number of rows of $A$ is equal to the number of columns of $B$.

(d) $A$ and $B$ have the same number of columns.

**Answer:**

To multiply two matrices $A$ and $B$, the **number of columns in matrix $A$** must be **equal to the number of rows in matrix $B$**.

This is a fundamental rule in matrix multiplication.

✅ **Correct Answer:**

**(b) the number of columns of A is equal to the number of rows of B**

2022

10. If $A = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $AI$ is equal to [1]

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 10 \\ -3 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} 15 & 15 \\ -1 & -1 \end{bmatrix}$

**Answer:**

We are given:

$$A = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We are asked to compute $AI$ (i.e., matrix $A$ multiplied by the identity matrix $I$).

---

### Property of Identity Matrix:

Multiplying any matrix $A$ by the identity matrix $I$ (on the right) leaves $A$ unchanged:

$$AI = A$$

So:

$$AI = \begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$$

---

### ✅ Correct Answer:

(c) $\begin{bmatrix} 5 & 10 \\ 3 & -4 \end{bmatrix}$

22. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$, then

$5A - BC$ is equal to [2]

(a) $\begin{bmatrix} -5 & -23 \\ 1 & 17 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 23 \\ 1 & 17 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 8 \\ -3 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$

**Answer:**

We are given:

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

We are to compute:

$$5A - BC$$

---

**Step 1: Compute $5A$**

$$5A = 5 \cdot \begin{bmatrix} 3 & 5 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix}$$

---

**Step 2: Compute $BC$**

$$B = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$$

Now multiply:

$$BC = \begin{bmatrix} (2 \times 1 + 4 \times 2) & (2 \times (-1) + 4 \times 1) \\ (0 \times 1 + 3 \times 2) & (0 \times (-1) + 3 \times 1) \end{bmatrix} = \begin{bmatrix} 2+8 & -2+4 \\ 0+6 & 0+3 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 6 & 3 \end{bmatrix}$$

**Step 3: Compute $5A - BC$**

$$5A = \begin{bmatrix} 15 & 25 \\ 5 & 20 \end{bmatrix}, \quad BC = \begin{bmatrix} 10 & 2 \\ 6 & 3 \end{bmatrix}$$

Subtract:

$$5A - BC = \begin{bmatrix} 15-10 & 25-2 \\ 5-6 & 20-3 \end{bmatrix} = \begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$$

---

✅ **Final Answer:**

(d) $\begin{bmatrix} 5 & 23 \\ -1 & 17 \end{bmatrix}$

2020

(b) Given $A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}$ [3]

If $A^2 = 3I$, where $I$ is the identity matrix of order 2, find $x$ and $y$.

**Answer:**

We are given:

$$A = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix}, \quad A^2 = 3I$$

Where $I$ is the **identity matrix of order 2**, so:

$$3I = 3 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

---

## Step 1: Compute $A^2 = A \cdot A$

$$A^2 = \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} \cdot \begin{bmatrix} x & 3 \\ y & 3 \end{bmatrix} = \begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix}$$

We are told:

$$A^2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

### Step 2: Equate elements

From the computed matrix:

$$A^2 = \begin{bmatrix} x^2 + 3y & 3x + 9 \\ xy + 3y & 3y + 9 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Now compare entries:

1. $x^2 + 3y = 3$
2. $3x + 9 = 0 \Rightarrow x = -3$
3. $xy + 3y = 0$
4. $3y + 9 = 3 \Rightarrow 3y = -6 \Rightarrow y = -2$

---

### Step 3: Verify values

Check equation (1):

$$x^2 + 3y = (-3)^2 + 3(-2) = 9 - 6 = 3 \quad \checkmark$$

Check equation (3):

$$xy + 3y = (-3)(-2) + 3(-2) = 6 - 6 = 0 \quad \checkmark$$

### ✅ Final Answer:

$$\boxed{x = -3,\ y = -2}$$

---

2019

**Question 3**

(a) Simplify [3]

$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

**Answer:**

We are given the expression:

$$\sin A\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}+\cos A\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}$$

---

### Step 1: Multiply scalar with matrices

Distribute the scalars:

**First matrix:**

$$\sin A\cdot\begin{bmatrix}\sin A & -\cos A\\ \cos A & \sin A\end{bmatrix}=\begin{bmatrix}\sin^2 A & -\sin A\cos A\\ \sin A\cos A & \sin^2 A\end{bmatrix}$$

**Second matrix:**

$$\cos A\cdot\begin{bmatrix}\cos A & \sin A\\ -\sin A & \cos A\end{bmatrix}=\begin{bmatrix}\cos^2 A & \cos A\sin A\\ -\cos A\sin A & \cos^2 A\end{bmatrix}$$

---

### Step 2: Add the two matrices

$$\begin{bmatrix}\sin^2 A & -\sin A\cos A\\ \sin A\cos A & \sin^2 A\end{bmatrix}+\begin{bmatrix}\cos^2 A & \sin A\cos A\\ -\sin A\cos A & \cos^2 A\end{bmatrix}=\begin{bmatrix}\sin^2 A+\cos^2 A & 0\\ 0 & \sin^2 A+\cos^2 A\end{bmatrix}$$

**Step 3: Use identity $\sin^2 A+\cos^2 A=1$**

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

---

✅ **Final Answer:**

$$\boxed{I\ \text{(the identity matrix)}}$$

---

(c) Given $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6\,I$, where M is a matrix and I is unit matrix of order 2x2. [4]

(i) State the order of matrix M.

(ii) Find the matrix M.

**Answer:**

We are given:

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I, \quad \text{where } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Let's solve each part step-by-step.

---

### (i) State the order of matrix M

We are multiplying a **2×2 matrix** with $M$, and getting a **2×2 result**. So, matrix multiplication is possible only if:

$$\text{Order of } M = 2 \times 2$$

✅ **Answer (i):** Order of matrix $M$ is **2×2**

---

### (ii) Find the matrix M

We are given:

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

## (ii) Find the matrix M

We are given:

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} M = 6I = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Let:

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Now multiply:

$$\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 4a+2c & 4b+2d \\ -a+c & -b+d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

---

## Step 1: Form equations

From:

$$\begin{aligned} 4a + 2c &= 6 \quad (1) \\ 4b + 2d &= 0 \quad (2) \\ -a + c &= 0 \quad (3) \\ -b + d &= 6 \quad (4) \end{aligned}$$

---

### Step 2: Solve

From (3):

$$-a + c = 0 \Rightarrow c = a \tag{3'}$$

From (4):

$$-b + d = 6 \Rightarrow d = b + 6 \tag{4'}$$

Substitute (3') into (1):

$$4a + 2a = 6 \Rightarrow 6a = 6 \Rightarrow a = 1, \ \Rightarrow c = 1$$

Substitute (4') into (2):

$$4b + 2(b + 6) = 0 \Rightarrow 4b + 2b + 12 = 0 \Rightarrow 6b = -12 \Rightarrow b = -2, \ \Rightarrow d = 4$$

---

### ✅ Final Answer (ii):

$$M = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$$

---

**2018**

**Question 1**

(a) Find the value of '$x$' and '$y$' if: [3]

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

**Answer:**

We are given the matrix equation:

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

---

### Step 1: Multiply the first matrix by 2

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} = \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix}$$

---

### Step 2: Add the matrices

$$\begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 2x+6 & 14-7 \\ 18+4 & 2y-10+5 \end{bmatrix} = \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix}$$

---

### Step 3: Set equal to the RHS matrix

$$\begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

Now equate corresponding elements:

1. $2x + 6 = 10 \Rightarrow 2x = 4 \Rightarrow x = 2$
2. $2y - 5 = 15 \Rightarrow 2y = 20 \Rightarrow y = 10$

---

### ✅ Final Answer:

$$\boxed{x = 2, \quad y = 10}$$

2018

(b) If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find $AC + B^2 - 10C$. [3]

**Answer:**

We are given:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

We are to compute:

$$AC + B^2 - 10C$$

---

### Step 1: Compute $AC$

$$A \cdot C = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$$

Multiply:

$$AC = \begin{bmatrix} (2)(1) + (3)(-1) & (2)(0) + (3)(4) \\ (5)(1) + (7)(-1) & (5)(0) + (7)(4) \end{bmatrix} = \begin{bmatrix} 2 - 3 & 0 + 12 \\ 5 - 7 & 0 + 28 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

---

### Step 2: Compute $B^2$

$$B^2 = B \cdot B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} \cdot \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$$

Multiply:

$$B^2 = \begin{bmatrix} (0)(0) + (4)(-1) & (0)(4) + (4)(7) \\ (-1)(0) + (7)(-1) & (-1)(4) + (7)(7) \end{bmatrix} = \begin{bmatrix} 0 - 4 & 0 + 28 \\ 0 - 7 & -4 + 49 \end{bmatrix} = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

### Step 3: Compute $10C$

$$10C = 10 \cdot \begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

---

### Step 4: Add and subtract:

$$AC + B^2 - 10C = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

First, add $AC + B^2$:

$$\begin{bmatrix} -1-4 & 12+28 \\ -2-7 & 28+45 \end{bmatrix} = \begin{bmatrix} -5 & 40 \\ -9 & 73 \end{bmatrix}$$

Then subtract $10C$:

$$\begin{bmatrix} -5-10 & 40-0 \\ -9-(-10) & 73-40 \end{bmatrix} = \begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}$$

---

### ✅ Final Answer:

$$\boxed{\begin{bmatrix} -15 & 40 \\ 1 & 33 \end{bmatrix}}$$

---

**2017**

(b) If A = $\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}$ and $A^2 - 5B^2 = 5C$. Find matrix C where C is a 2 by 2 matrix. [4]

**Answer:**

We are given:

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix}, \quad A^2 - 5B^2 = 5C$$

We are to find matrix $C$.

---

**Step 1: Compute $A^2 = A \cdot A$**

$$A^2 = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 3 \cdot 3 & 1 \cdot 3 + 3 \cdot 4 \\ 3 \cdot 1 + 4 \cdot 3 & 3 \cdot 3 + 4 \cdot 4 \end{bmatrix} = \begin{bmatrix} 1+9 & 3+12 \\ 3+12 & 9+16 \end{bmatrix} = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix}$$

---

**Step 2: Compute $B^2 = B \cdot B$**

$$B^2 = \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} (-2)(-2) + (1)(-3) & (-2)(1) + (1)(2) \\ (-3)(-2) + (2)(-3) & (-3)(1) + (2)(2) \end{bmatrix} = \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

---

**Step 3: Compute $A^2 - 5B^2$**

$$5B^2 = 5 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$A^2 - 5B^2 = \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix}$$

---

### Step 4: Solve for $C$

We are given:

$$A^2 - 5B^2 = 5C \Rightarrow C = \frac{1}{5}(A^2 - 5B^2) = \frac{1}{5}\begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$$

---

### ✅ Final Answer:

$$\boxed{C = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}}$$

---

2017

**Question 5**

(a) Given matrix $B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}$. Find the matrix X if, $X = B^2 - 4B$. [4]

Hence solve for $a$ and $b$ given $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

**Answer:**

We are given:

$$B = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix}, \quad X = B^2 - 4B$$

Also:

$$X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

We need to:

1. Find $X$
2. Solve for $a$ and $b$

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### Step 1: Compute $B^2 = B \cdot B$

$$B^2 = \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 1 \cdot 8 & 1 \cdot 1 + 1 \cdot 3 \\ 8 \cdot 1 + 3 \cdot 8 & 8 \cdot 1 + 3 \cdot 3 \end{bmatrix} = \begin{bmatrix} 1 + 8 & 1 + 3 \\ 8 + 24 & 8 + 9 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix}$$

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### Step 2: Compute $4B$

$$4B = 4 \cdot \begin{bmatrix} 1 & 1 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix}$$

### Step 3: Compute $X = B^2 - 4B$

$$X = \begin{bmatrix} 9 & 4 \\ 32 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 32 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

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### Step 4: Solve $X \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5a \\ 5b \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \end{bmatrix}$$

So,

- $5a = 5 \Rightarrow a = 1$
- $5b = 50 \Rightarrow b = 10$

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### ✅ Final Answer:

- Matrix $X = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$
- $a = 1, \quad b = 10$

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