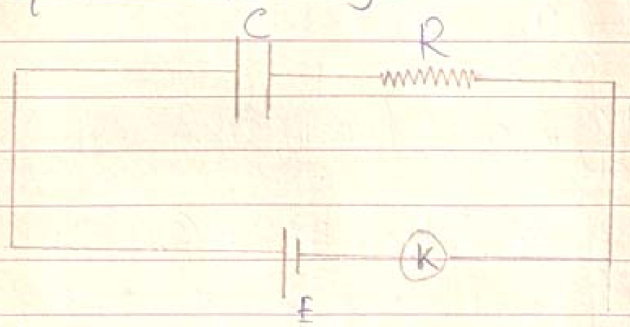
C-R Circuit :

(i) Charging of a condenser through a resistance i.e Growth of charge :-



A Capacitor is to be charged through a resistance, by a steady source of e.m.f. When the key 'K' is closed the charge flows from the source to the plates of the capacitor & the P.d across the capacitor rises. The flow of charge continues till the P.d across the capacitor becomes equal to the e.m.f of the source. Thus as long as charge flows it gives current in the circuit. When the capacitor gets fully charged, the current in the circuit becomes zero.

Given:

C = Capacitance of the Capacitor.

R = Resistance of the circuit.

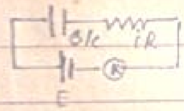
Q_0 = Final or max^m charge stored in the capacitor

E = E.m.f of the source.

Let Q = Charge stored in the capacitor at any instant t, during charging.

Let i = current flowing in the circuit at that instant of time 't'.

The e.m.f eq^n of the circuit :



$\frac{Q}{C} + iR = E$ But $i = \frac{dQ}{dt}$

$R\frac{dQ}{dt} + \frac{Q}{C} = E$ — (1)

when $\boxed{Q = Q_0}$ $i = \frac{dQ}{dt} = 0$ from (1)

$\therefore$ $R \cdot 0 + \frac{Q_0}{C} = E$ or $Q_0 = EC$ — (2)

Eq^n (1) Can be re-arranged as

$$\frac{dQ}{dt} = -\frac{1}{RC}\left[Q - EC\right]$$

putting eq^n (2) : $\frac{dQ}{dt} = \frac{-1}{RC}\left[Q - Q_0\right]$

or $\frac{dQ}{(Q - Q_0)} = -\frac{dt}{RC}$ Integrating both sides

$\log(Q - Q_0) = -\frac{t}{RC} + A$ — (3) where A is unknown constant of integration

At $t = 0$; $Q = 0$ from (3) : $A = \log(-Q_0)$ — (4)

putting eq^n (4) in (3) :-

$$\log(Q - Q_0) = -\frac{t}{RC} + \log(-Q_0)$$

or $\log\frac{(Q - Q_0)}{(-Q_0)} = -\frac{t}{RC}$

or $\log\left(1 - \frac{Q}{Q_0}\right) = -\frac{t}{RC}$ taking anti-log

$1 - \frac{Q}{Q_0} = e^{-t/RC}$ or $\frac{Q}{Q_0} = 1 - e^{-t/RC}$

or $\boxed{Q = Q_0\left(1 - e^{-t/RC}\right)}$ — ⑤

Using eqⁿ ⑤, the charge stored in the capacitor at any instant 't' can be calculated.
The current flowing in the circuit at the instant 't' during charging can be obtained by differentiating eqⁿ ⑤.

$$i = \frac{dQ}{dt} = Q_0\left[0 - \left(-\frac{1}{RC}\right)e^{-t/RC}\right]$$

$$\text{or } i = \frac{Q_0}{RC}e^{-t/RC}$$

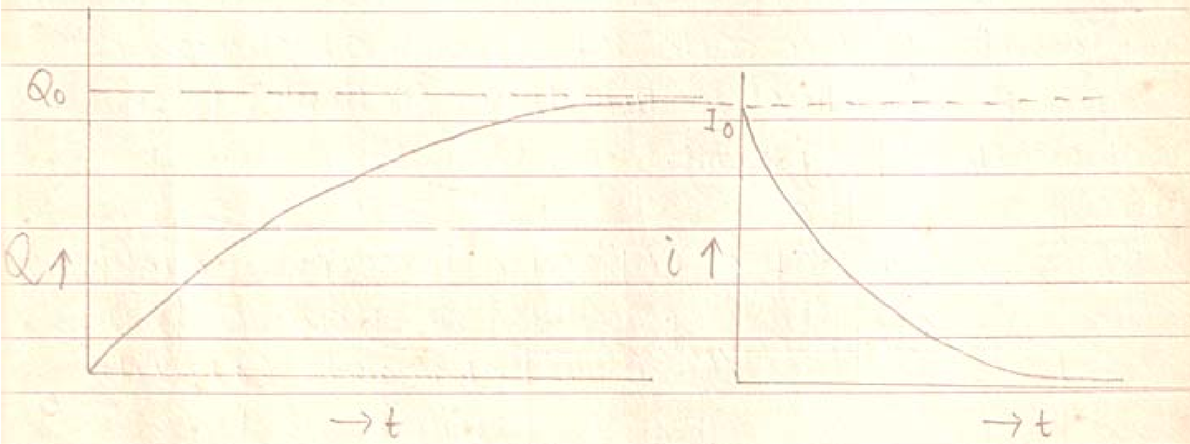
put $\frac{Q_0}{RC} = I_0$

or $\boxed{i = I_0 e^{-t/RC}}$ — ⑥

Dimension of $\frac{Q_0}{RC} = \frac{CV}{RC} = \frac{V}{R} = i$ = dimension of current $\therefore \frac{Q_0}{RC} = I_0$ (?)

Discussions: ① from eqⁿ ⑤ we find that

$Q = Q_0$, when $t = \infty$ & from ⑥ at $t = \infty, i = 0$

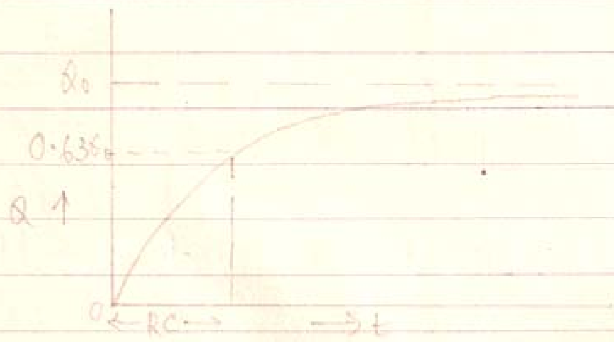


Thus the growth of charge is exponential and asymptotic to the line $Q = Q_0$. The moment charge starts growing, the current in the circuit decreases from max^m value I_0 in an exponential fashion and becomes asymptotic to the line $i = 0$

At $t = RC$ = const.

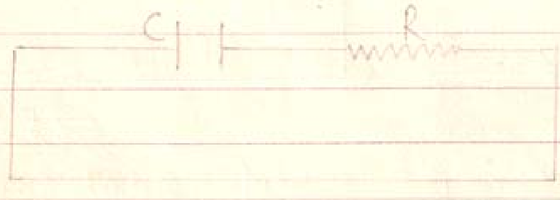
from (5) ; $Q = Q_0 [1 - e^{-1}] = 0.63 Q_0$

Hence the const. $t = RC$ is known as Capacitive time constant of the circuit and can be defined as the time during which the charge grows to 63% of its max^m value.



Physical Significance : Time const. gives an idea about the rate of growth of charge in the circuit. Smaller is the time constant, faster is the rate of growth.

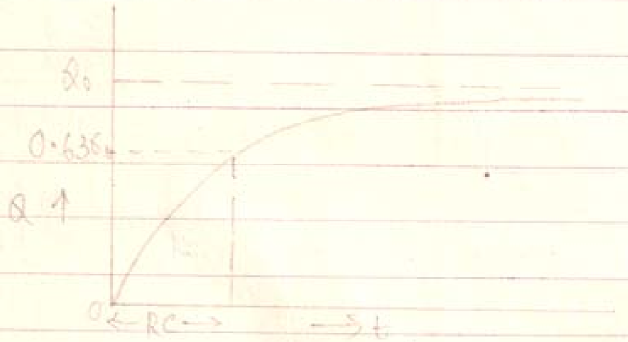
(ii) Discharge of a Capacitor through a resistance:



We have a fully charged capacitor which is to be discharged through asymptotic to the line $Q = Q_0$. The moment charge starts growing, the current in the circuit decreases from max^m value I_0 in an exponential fashion and becomes asymptotic to the line $i = 0$

At $t = RC = $ const.
from ⑤ ; $Q = Q_0 [1 - e^{-1}] = 0.63 Q_0$

Hence the const. $t = RC$ is known as capacitive time constant of the circuit and can be defined as the time during which the charge grows to 63% of its max^m value.



Physical Significance : Time const. gives an idea about the rate of growth of charge in the circuit. Smaller is the time constant, faster is the rate of growth.

(ii) Discharge of a Capacitor through a resistance :

We have a fully charged capacitor which is to be discharged through a resistance, by joining the two plates of the Capacitor. During the discharge a momentary current flows through the circuit.

Let Q_0 = Initial charge stored in the Capacitor.

Q = charge stored in the capacitor at any instant 't' during discharge.

i = current flowing in the circuit at that instant 't'.

The e.m.f eq^n of the circuit

$$\frac{Q}{C} + iR = 0 \quad \text{put } i = \frac{dQ}{dt}$$

$$R.\frac{dQ}{dt} = -\frac{Q}{C} \quad \text{or} \quad \frac{dQ}{Q} = -\frac{dt}{RC}$$ - Integrating both sides

$$\log Q = -\frac{t}{RC} + B \quad \text{— (7)}$$ where B is unknown Const. of integration.

Applying initial cond^n at $t=0$, $Q = Q_0$

$$\log Q_0 = -\frac{0}{RC} + B \quad \text{or} \quad B = \log Q_0 \quad \text{— (8)}$$

putting eq^n (8) in (7) $\log \frac{Q}{Q_0} = -\frac{t}{RC}$

or $Q = Q_0 e^{-t/RC}$ — (9)

Using eq^n (9) we can find the charge stored in the Capacitor at instant 't' during discharge.

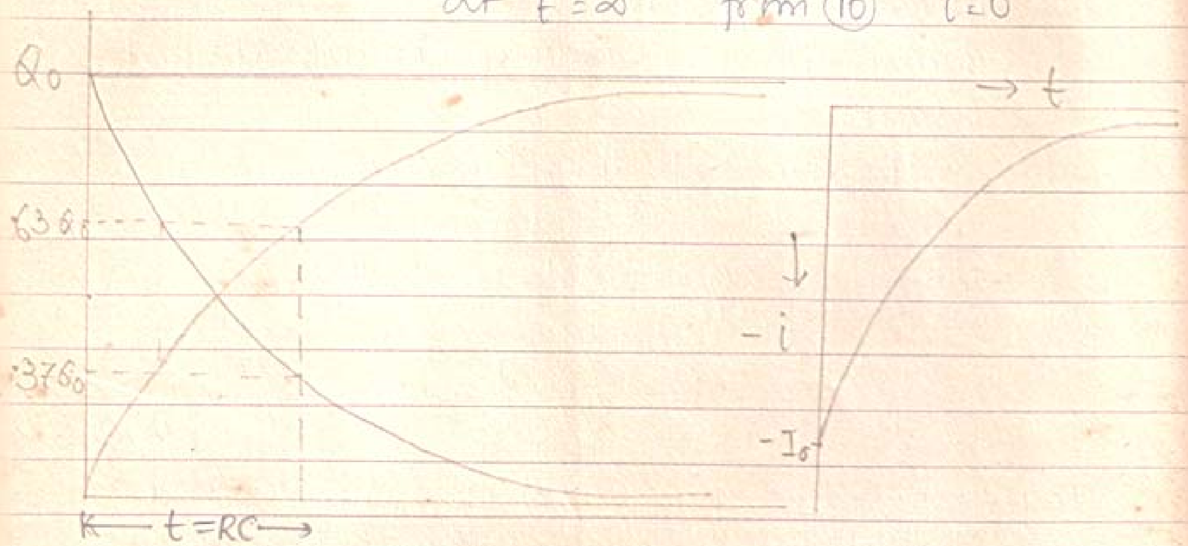
The current flowing in the circuit, at the instant 't' during discharge:

$$i = \frac{dQ}{dt} = -\frac{Q_0}{RC} e^{-t/RC}$$

put $\frac{Q_0}{RC} = I_0 = \text{Const.}$ $\therefore i = -I_0 e^{-t/RC}$ — (10)

The negative sign indicates that the current flowing in the circuit is in opposite direction to that during charging.

Discussions: (1) at $t = \infty$; from (9) $Q = 0$
at $t = \infty$ from (10) $i = 0$



(2). at $t = RC$ = Capacitive time const, from (9) we get $Q = Q_0 e^{-1} = .37 Q_0$.
Hence Capacitive time const. can also be defined as the time, during which the charge falls 37 % of its maximum value.

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R.U.'86. Investigate under what conditions the discharge of a condenser through an electric circuit containing an inductance and resistance will be oscillatory in character. Find the period of oscillation.

R.U.'82 Discuss the discharge of a capacitance C through an inductance L and a resistance R in series when does it become oscillatory? (discharge L-C-R.)