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Dielectric Sphere in a Uniform Field:

Given: a = Radius of the dielectric sphere.

ϵ_1 and ϵ_2 = Permitivity of the dielectric sphere and the medium respectively.

E_0 = The uniform dielectric field existing in the medium.

Since the dielectric sphere is placed in uniform electric field and there is no free charge and inside and outside the surface hence Potential must satisfy Laplace's equation.

(i) $\nabla^2 V_1 = 0$; $\nabla^2 V_2 = 0$

(ii) Potential at a point $P(r, \theta)$

$V_1 = V_2$ when $r = a$

i.e $V(r, \theta)$ must be continuous.

(iii) The normal component of $\vec{D}$ must be continuous on $r = a$ for all θ

i.e $$\epsilon_1 \frac{\partial V_1}{\partial r} = \epsilon_2 \frac{\partial V_2}{\partial r} \quad \text{on } r = a$$

(iv) $V_1(r, \theta)$ must be finite at $r = 0$

(v) $V_2(r, \theta) = V_0 = -E_0 z = -E_0 r \cos\theta$ as $r \to \infty$

The potential which is the solution of the Laplace's equation can be expressed in the form:

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$$V = \sum_{l=1}^{\infty} \left[A_l r^l + B_l r^{-(l+1)} \right] P_l \cos\theta \quad \text{--- (1)}$$

Since the potential at the origin must be finite no term of the form $B r^{-(l+1)}$ should be included in the expression for V_1, as it gives infinite contribution to the potential at $r=0$. Therefore

$$V_1 = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \quad \text{--- (2)}$$

The potential outside the sphere i.e. V_2 can be given by

$$V_2 = \sum_{l=0}^{\infty} \left[C_l r^l + D_l r^{-(l+1)} \right] P_l \cos\theta \quad \text{--- (3)}$$

The constants can be formed from the boundary conditions.

The potential V_2 at infinity is

$$V_2 = C_0 P_0(\cos\theta) + C_1 r P_1(\cos\theta) + C_2 r^2 P_2(\cos\theta) + \cdots \quad \text{--- (4)}$$

But from boundary condition (V) potential at infinity must be equal to $-E_0 r P_1 \cos\theta$

Equating the coeff. of (4) with (5) we find all term vanishes except 2nd term

$l = 1$; $\boxed{C_1 = -E_0}$

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The boundary condition (ii) gives

$$V_1 = V_2 \quad \text{on } r = a \text{ for all } \theta.$$