Derivation of PLANKS Law (By the application of B-E statistics):

Consider a black body enclosure of volume V at T kelvin in thermal equilibrium. The enclosure contains photons of different energies and these move about in all possible directions.

The photons are subject to the following conditions:

(i) The rest mass of photon is zero.

(ii) During emission, new photons from the atoms are created so that

$$\sum_i \delta n_i \neq 0 \quad \text{\& therefore} \quad \alpha = 0$$

(iii) The modes of propagation of photon are mutually perpendicular to each other as radiations may be polarized in two ways.

Consider an element of volume $d\tau$ in phase space. The subject to Heisenberg's uncertainty principle we have

$$d\tau = dx \cdot dy \cdot dz \cdot dp_x \cdot dp_y \cdot dp_z = h^3 \quad \text{—(1)}$$

Where, h = Plank's constant.
$dx \cdot dy \cdot dz$ = Position parameters.
$dp_x\, dp_y\, dp_z$ = Momentum Parameters.

Now, let the phase space be split into the position and momentum spaces and let the position space be extended to cover the whole enclosure. Then the particle (photon) may be present any where in the enclosure and the element of volume in the momentum space of size σ_p is given by

$$\sigma_p = \frac{h^3}{V} \quad \text{—(2)}$$

Obviously σ_p gives one value of momentum. Then the no. of momenta contained in the volume of the shell between two concentric spheres of radii p & $p+dp$ will be given as

$$\frac{4}{3}\pi\left[(p+dp)^3 - p^3\right]/\sigma_p$$

$$= \frac{4}{3}\pi\left\{p^3 + 3p^2 dp - p^3\right\}/\sigma_p$$

Neglecting dp^2 and higher powers.

Thus,

$$\text{No. of momenta} = 4\pi p^2\, dp \cdot \frac{V}{h^3}$$

$$= \frac{4\pi p^2 V dp}{h^3} \quad \Big| \text{using eq}^n \text{ (2)}$$

In view of assumption (iii) this no. must be doubled. Then

$$\text{No. of momenta} = \left(\frac{8\pi p^2}{h^3} \cdot V dp\right)$$

So that no. of momenta per unit volume is given by;

$$g(p)\,dp = \left\{\left(\frac{8\pi p^2}{h^3} V dp\right)\Big/ V\right\}$$

Which is also no. of photons.

Thus:

$$g(p)\,dp = \left(\frac{8\pi p^2}{h^3} dp\right) \quad \text{--- (3)}$$

Where $g(p)\,dp$ denotes the no. of particles per unit volume of momentum space considered between momenta 'p' & 'p+dp'.

Again,

$$p = \text{momentum of photon} = \left(\frac{h\nu}{c}\right) \quad \text{--- (4)}$$

Where, ν = frequency of radiation.

& c = velocity of light in vacuum

Then,

$$dp = \frac{h}{c}\,d\nu \quad \text{--- (5)}$$

So that eqn (3) may be transformed into a frequency eqn by using eqn (4) and (5) in eqn (3). Thus writing $g(\nu)\,d\nu$ for $g(p)\,dp$ we have

$$g(\nu)\,d\nu = \frac{8\pi}{h^3} \cdot \frac{h^2\nu^2}{c^2} \cdot \frac{h}{c}\,d\nu$$

$$\therefore\ g(\nu)\,d\nu = \frac{8\pi}{c^3}\,\nu^2\,d\nu \quad \text{--- (6)}$$

Where now

$g(\nu)\,d\nu$ gives the no. of photons per per unit volume.

The B-E distribution law is as follows:

$$dn = \frac{g(\nu)\,d\nu}{(e^{\alpha+\beta u}-1)} \quad \text{—(7)}$$

Where,

dn = no. of particles per unit volume

$$\left.\begin{aligned} \alpha &= \text{constant} = 0 \\ \beta &= \text{constant} = \frac{1}{kT} \end{aligned}\right\} \text{—(8)}$$

& $u = h\nu$

From eq^n (7) and eq^n (8) we get

$$dn = \left\{\frac{8\pi\nu^2 d\nu}{c^3} \cdot \frac{1}{(e^{h\nu/kT}-1)}\right\}$$

ie $$dn = \left\{\frac{8\pi\nu^2}{c^3} \cdot \frac{d\nu}{(e^{h\nu/kT}-1)}\right\} \quad \text{—(9)}.$$

Now,

The energy density $[E(\nu)\,d\nu]$ in the black body radiation considered between the frequency range ν and $(\nu+d\nu)$ is given by

$E(\nu)\,d\nu$ = Energy per unit volume of radiation in the range ν to $\nu+d\nu$

$\therefore E(\nu)\,d\nu = \{$(No. of photons per unit volume) × (the average energy per photon)$\}$

using eq^n (9) we obtain:

$$E(\nu)\,d\nu = \left[\left\{\frac{8\pi\nu^2 d\nu}{c^3(e^{h\nu/kT}-1)}\right\} \times h\nu\right]$$

Hence,

$$\boxed{E(\nu)\,d\nu = \left[\left(\frac{8\pi h\nu^3}{c^3}\right) \cdot \frac{d\nu}{(e^{h\nu/kT}-1)}\right]} \quad \text{—(10)}$$

Eq^n (10) is Plank's law in terms of frequency of radiation.