Bohr's Theory Of Hydrogen atom :-

Bohr applied, Plank's quantum ideas to the Rutherford's model of atom and made the following assumptions :-

Postulates :-

1. An atom consist's of a positively charged nucleus round which electrons rotate in circular orbits. The mass of the nucleus is very large compared to the mass of the electron, So that the nucleus can be assumed to be at rest.

2. The centripetal force required to keep the electron in uniform circular motion, is supplied by the Coulomb's electrostatic force between the positively charged nucleus and -vely charged electron.

3. All classically possible orbits are not allowed to the electrons, electrons can rotate in few discrete orbits, such that the orbital angular momentum of the electron is

$$I\omega = n\hbar = n\frac{h}{2\pi} \quad \text{where } n = 1, 2, 3 \ldots \text{ an integer}$$

and h is Plank's Const $h = 6.625 \times 10^{-34}$ J-Sec.

$\hbar = 1.05 \times 10^{-34}$ J-Sec.

4. According to classical theory, an accelerated charged particle emits radiation. Since the electron rotating in the orbit is a charged

particle in acelarated motion, it should emit radiation energy and the radius of its orbit should decrease, spiraling down to the nucleus. Thus according to classical theory no stable structure of atom can be conceived. Bohr assumed that as long as electron continues to rotate in the circular orbit arround the nucleus, it does not radiate any energy, inspite of being an acelarated motion. The circular orbits with a definite energy were called as Stationary Energy State.

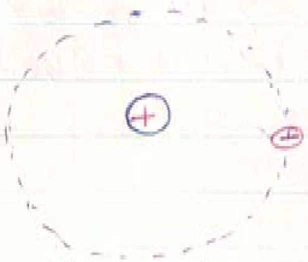
5. When an electron jumps from higher energy orbit to lower energy orbit, the difference in energy value of the two orbits is radiated in the form of light having frequency 'ν' given by $\boxed{h\nu = E_n - E_p}$ — (A)

E_n = Energy of the nth orbit from which the electron jumps.

E_p = Energy of the pth orbit to which the electron jumps.

Equation (A) is known as Bohr's frequency condition.

Hydrogen atom : (Generalised Case):



m = mass of the electron.
e = charge of the electron.
r_n = radius of the nth Circular orbit.
v_n = Speed of the electron in the nth circular orbit.

Z = Atomic number of the element.

Applying 2nd Postulates:-

$$\frac{m v_n^2}{r_n} = \frac{(Ze).e}{4\pi\epsilon_0 . r_n^2} \quad \text{——} \; (1)$$

$$\text{or} \quad m r_n v_n^2 = \frac{Ze^2}{4\pi\epsilon_0} \quad \text{——} \; (2)$$

Applying the 3rd postulate:

$$\text{Angular momentum} = I\omega = m r_n^2 . \frac{v_n}{r_n} = \frac{nh}{2\pi}$$

$$\text{Or} \quad m r_n v_n = \frac{nh}{2\pi} \quad \text{——} \; (3)$$

Squaring eq^n (3) & dividing it by eq^n (2):-

$$\frac{m^{\cancel{2}} r_n^{\cancel{2}} \cancel{v_n^2}}{\cancel{m}\,\cancel{r_n}\,\cancel{v_n^2}} = \frac{\dfrac{n^2h^2}{4\pi^2}}{\dfrac{Ze^2}{\cancel{4\pi}\epsilon_0}}$$

$$r_n = \frac{\epsilon_0}{\pi} \frac{n^2h^2}{Zme^2} \quad \text{——} \; (4)$$

Eq^n (4) gives the radius of the Bohr's orbit.

Calculate the radius of the 1st Bohr orbit for hydrogen atom:-

$$r_0 = \frac{8.852 \times 10^{-12} \times 1 \times (6.625 \times 10^{-34})^2}{3.172 \times 1 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 8.3 \times 10^{-9} \text{ cm.}$$

We now find the energy of the electron in the nth orbit:-

The P.E of the electron V_n = Charge x Potential

$$\text{or } V_n = \frac{(Ze) \cdot (-e)}{4\pi\epsilon_0 r_n} = -\frac{Ze^2}{4\pi\epsilon_0 r_n}$$

The K.E of the electron:

$$T_n = \frac{1}{2} m v_n^2$$

$$T_n = \frac{1}{2} \frac{Ze^2}{4\pi\epsilon_0 r_n}$$

Total Energy of the electron in the nth orbit

$$E_n = T_n + V_n = \frac{Ze^2}{8\pi\epsilon_0 r_n} - \frac{Ze^2}{4\pi\epsilon_0 r_n} = -\frac{Ze^2}{8\pi\epsilon_0 r_n}$$

$$\boxed{E_n = -\frac{Ze^2}{8\pi\epsilon_0 r_n}} \quad \text{—} \; (5)$$

[from eqn (5) we find that the total energy of the electron in the orbit is -ve i.e less than zero. As the radius of the orbit increases the energy becomes less & less -ve i.e energy increases and when $r_n = \infty$; $E_n = 0$ i.e when the electron is detached from the atom and becomes free electron its energy becomes maximum equal to zero. Thus the negative energy indicates that the electron in the orbit moves in favour of the force.]

Putting eq^n (4) in (5) :-

$$E_n = -\frac{1}{8\pi\epsilon_0} \cdot \frac{Ze^2}{\frac{\epsilon_0 n^2 h^2}{\pi Z m e^2}}$$

$$\boxed{E_n = -\frac{1}{8\cancel{\pi}\epsilon_0^2} \frac{Z^2 m e^4}{n^2 h^2}} \quad \text{—— (6)}$$

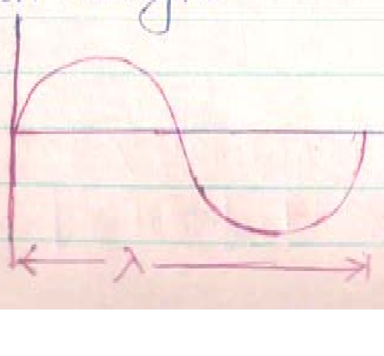
Let an electron jump from nth orbit to pth orbit the frequency of radiation is given by

$$h\nu = E_n - E_p = -\frac{1}{8\epsilon_0^2} \cdot \frac{Z^2 m e^4}{n^2 h^2} - \left(-\frac{1}{8\epsilon_0^2} \frac{Z^2 m e^4}{p^2 h^2}\right)$$

$$\nu = \frac{1}{8\epsilon_0^2} \frac{Z^2 m e^4}{h^3} \left[\frac{1}{p^2} - \frac{1}{n^2}\right] \quad \text{—— (7)}$$

let $\bar{\nu}$ be the wave number of the emitted radiation.

[wave number : The number of waves per unit length



In λ length no. of waves = 1.

In 1 " " " " $\frac{1}{\lambda}$.

wave no. $\bar{\nu} = \frac{1}{\lambda}$ $\boxed{c = \nu\lambda}$

'84. Explain the Rydberg constant and screening factor of K-Shell for copper (Z = 29). If the K-sh are respectively 135 eV and 8940 eV.

26/2/90

$\bar{\nu} = \frac{\nu}{c}$, from ⑦ dividing by c

$$\bar{\nu} = \frac{z^2 m e^4}{8 \epsilon_0^2 C h^3}\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \quad — ⑧$$

Since m, e, ϵ_0, c, h are all constants

[.A.S'89

put $\frac{me^4}{8\epsilon_0^2 ch^3}$ = R = Rydberg's Constant.

$$R = \frac{9.1 \times 10^{-31} kg \times (1.6 \times 10^{-19} coulomb)^4}{8 \times (8.862 \times 10^{-12} f/m)^2 \times (3 \times 10^8 m/sec)(6.625}$$

$$R = 1.09 \times 10^7 \ m^{-1}$$

$$\therefore \bar{\nu} = Rz^2\left[\frac{1}{p^2} - \frac{1}{n^2}\right] \quad — ⑨$$

Stationary States: Energy of the electron in the nth orbit from ⑥ $E_n = -\frac{Rz^2}{n^2}$

$$\boxed{E_n = -\frac{13.6 \ z^2}{n^2} \ e.v} \quad — ⑩$$

[Electron volt is a smaller unit of energy. It is defined as the work done in taking one electron through one volt P.d.

W = Charge × P.D

1 e.v = 1.6×10^{-19} coulomb × 100 volt

1 e.v = 1.6×10^{-19} Joules]

Since the energy of an electron in an orbit remains constant throughout the motion, the orbits are therefore known as stationary states and can be represented diagramatically by straight line ie a levelled diagram known as energy-level diagram.