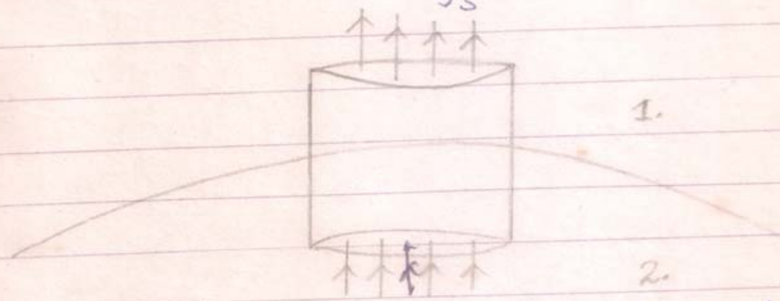
Boundary conditions in Magnetostatics:

There are two boundary conditions at the interface between two media of magnetic permiabilities μ_1 & μ_2.

1. From Gauss's divergence theorem

$$\int_S \vec{B}\cdot\hat{e}_n\,ds = \int_\tau \nabla\cdot\vec{B}\,d\tau \quad \text{—(1)}$$

$$\because\ \nabla\cdot\vec{B} = 0 \qquad \int_S \vec{B}\cdot\hat{e}_n\,ds = 0 \quad \text{—(2)}$$



Total flux through the two faces

$$\int_S \vec{B}\cdot\hat{e}_n\,ds = \int_S \vec{B}_1\cdot\hat{e}_n\,ds + \int_S -\vec{B}_2\cdot\hat{e}_n\,ds \quad \text{—(3)}$$

negative sign indicates the ingoing and outgoing direction of lines of force.

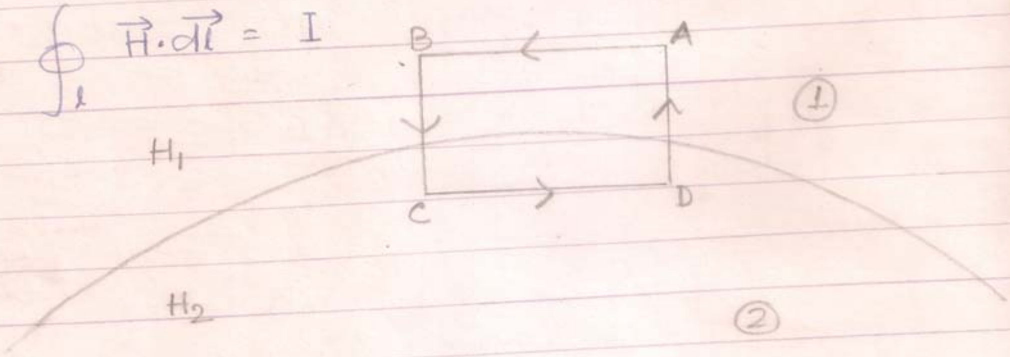
$$\int_S \vec{B}_1\cdot\hat{e}_n\,ds = \int_S \vec{B}_2\cdot\hat{e}_n\,ds$$

$$\text{or} \quad (B_1)_\perp = (B_2)_\perp$$

Thus the normal components of the magnetic induction vector are same on the two sides of the boundary.

2. From Ampere's circuital law

$$\oint_l \vec{H}\cdot\vec{dl} = I$$



Let $AB = CD = dl$

BC and AD are vanishingly small.

Sides AB and CD are chosen to be parallel to the surface.

$$\int \vec{H}\cdot\vec{dl} = \int_{AB} \vec{H}\cdot\vec{dl} + \int_{BC} \vec{H}\cdot\vec{dl} + \int_{CD} \vec{H}\cdot\vec{dl} + \int_{DA} \vec{H}\cdot\vec{dl}$$

The line integral of $\vec{H}$ over the lengths BC & DA are zero.

The negative sign is to be used as the direction of $\vec{AB}$ and $\vec{CD}$ being opposite

$$\oint \vec{H}\cdot\vec{dl} = \int \vec{H_1}\cdot\vec{dl} + \int -\vec{H_2}\cdot\vec{dl} = I$$

or $(H_1)_{\parallel} - (H_2)_{\parallel} = I$ if $I = 0$ $\therefore (H_1)_{\parallel} = (H_2)_{\parallel}$

Thus the tangential components of the magnetic field strength $\vec{H}$ are the Same On the two Sides of the boundary.