Qr②. What is the advantage of an active filter over passive filter? Can an unity gain amplifier be used in a low pass filter? Describe Butterworth active low pass filter along with its Charecteristic. [636]

Active filters are filters that employ passive elements, usually Registors and Capacitors in conjunction with active elements such as Operational amplifier.

Advantages of active filters:

(1) As active filters contain no inductors it is possible to integrate them

& (2) they can be made to work at very low frequencies (fractions of hertz) where it is impractical to use Inductors.

(3) They have single ended inputs and outputs and thus do not 'float' with respect to the system power supply.

④. Amplifiers used for active elements have a limited input and output voltage range (± 10 V for most OP-AMP circuits). (adv or dis?)

Commercially available OP-AMPS have unity gain band-width, products as high as 100 MHZ, it is possible to design active filters up to frequencies of Several MHZ. The factor which matters for full wave response at high frequencies is the slewing rate of the OP-AMP. (Commercially Integrated OP-AMPS are available with slewing rate as high as 100 V/μs).

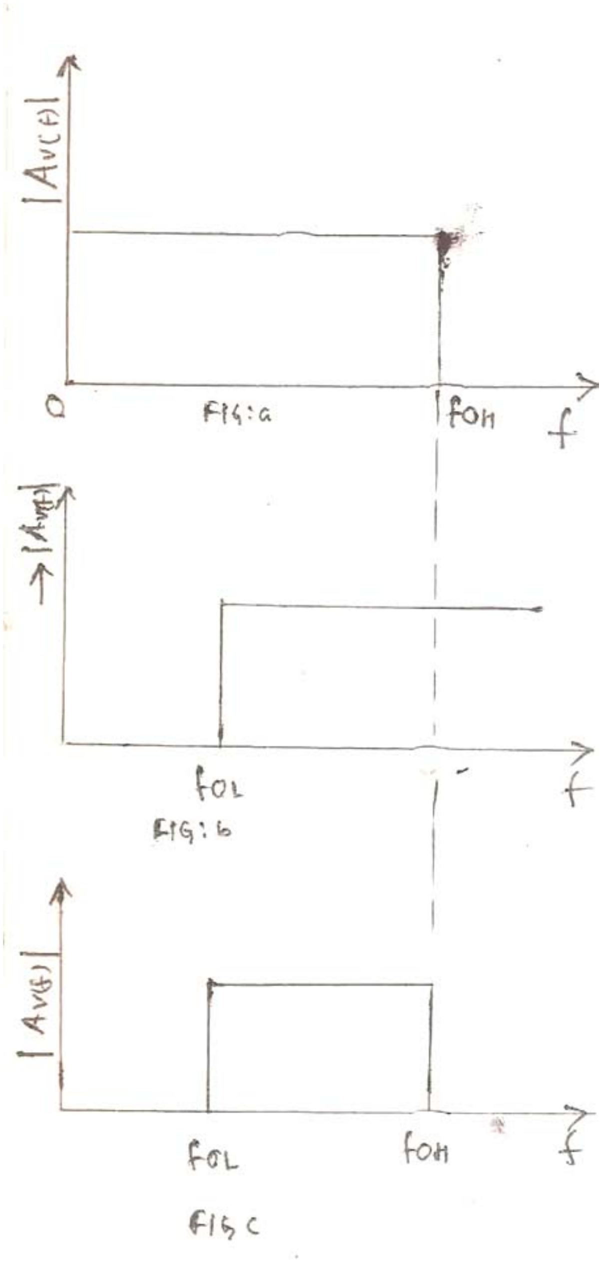


FIG:1.

Active filters: In an ideal low pass filter when all signals are within the band $0 \le f \le f_0$ are applied its response shows fig① that they are transmitted without loss. where as when input frequency[$(f) > f_0$] it gives zero output. But such ideal characteristic can not be obtained with physical elements.

Hence an approximation for an ideal low pass filter is made; which is of the form

$$A_v(s) = \frac{1}{P_n(s)} \quad \text{—— ①}$$

where,

$P_n(s)$ is a polynomial in the variable 'S' with zeros in the left hand plane. [* Active filters permit the realization of arbitrary left hand poles for $A_v(s)$, using the operational amplifier as active element and only Resistors and Capacitors for passive elements]

Butterworth filter: when eq^n ① is approximated using Butterworth polynomials $B_n(s)$ [with $s = j\omega$] we get

$$A_v(s) = \frac{A_{vo}}{B_n(s)} \quad \text{—— ②}$$

also,

$$|A_v(s)|^2 = |A_v(s)|\,|A_v(-s)| = \frac{A_0^2}{|B_n(\omega)|^2} \quad \text{—— ③}$$

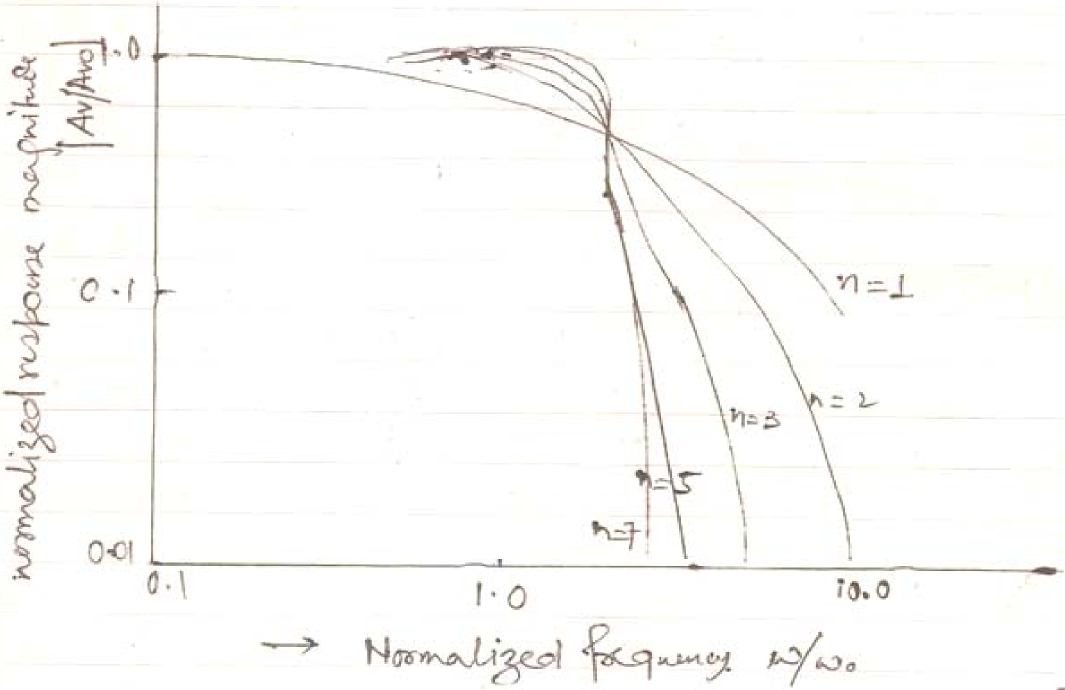
The magnitude of $B_n(\omega)$ is given by

$$|B_n(\omega)| = \sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^{2n}} \quad \text{—— ④}$$

Putting eq^n ④ in ③ we get

$$|A_v(s)|^2 = |A_v(s)||A_v(-s)| = \frac{A_{v_0}^2}{\left[1+\left(\frac{\omega}{\omega_0}\right)^{2n}\right]} \quad \text{—⑤}$$

For various values of 'n', the Butterworth response is plotted in fig.②:



FIG②

From the graph it is evident that the magnitude of A_v is down 3dB at $\omega = \omega_0$ for all n. The larger the value of n, the more closely the curve approximates the ideal low pass response.

We have the following Butterworth polynomials for different n values (assuming normalization & ω_0 = 1 rad)

when $n = 1$ $\quad P_n(s) = (s+1)$

when $n = 2$ $\quad P_n(s) = (s^2 + 1.414s + 1)$

when $n = 3$ $\quad P_n(s) = (s+1)(s^2+s+1)$

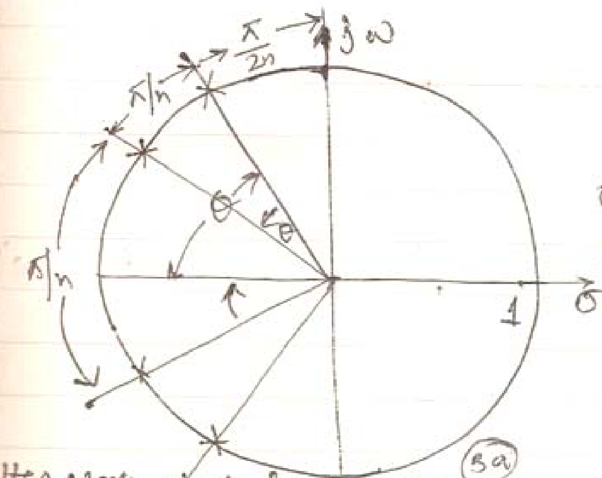
when $n = 4$ $\quad P_n(s) = (s^2 + 0.765s + 1)(s^2 + 1.848s + \ldots$ —⑥

Thus it is found that for even values of 'n' the polynomials are product of Quadratic forms & for odd values of 'n' there is additional factor (S+1).

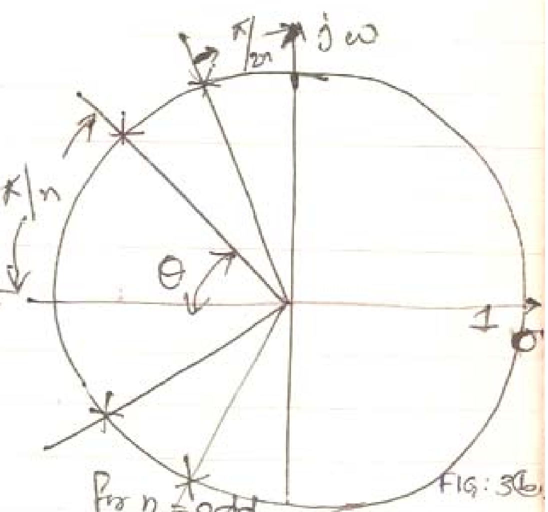
The zeros of the normalized Butterworth polynomials are either −1 or complex conjugate and are found on the so called Butterworth circle of unit radius.

The damping factor (K) is defined as one half the coefficient of s in each quadratic factor.

Mathematically : $K = \cos\theta$ — (7).



Butterworth circle for n = even (3a)



For n = odd FIG: 3(b)

from (2) : $\frac{A_v(s)}{A_{v_0}} = B_n(s)$, from the table the typical 2nd order Butterworth filter transfer is of the from

$$\frac{A_v(s)}{A_{v_0}} = \frac{1}{\left(S/\omega_0\right)^2 + 2k\left(\frac{S}{\omega_0}\right) + 1} \quad \text{— (8)}$$

where, $\omega_0 = 2\pi f_0$ is the high frequency 3-dB point. Similarly the 1st order filter is

$$\frac{A_v(s)}{A_{v_0}} = \frac{1}{\left(S/\omega_0 + 1\right)} \quad \text{— (9)}$$

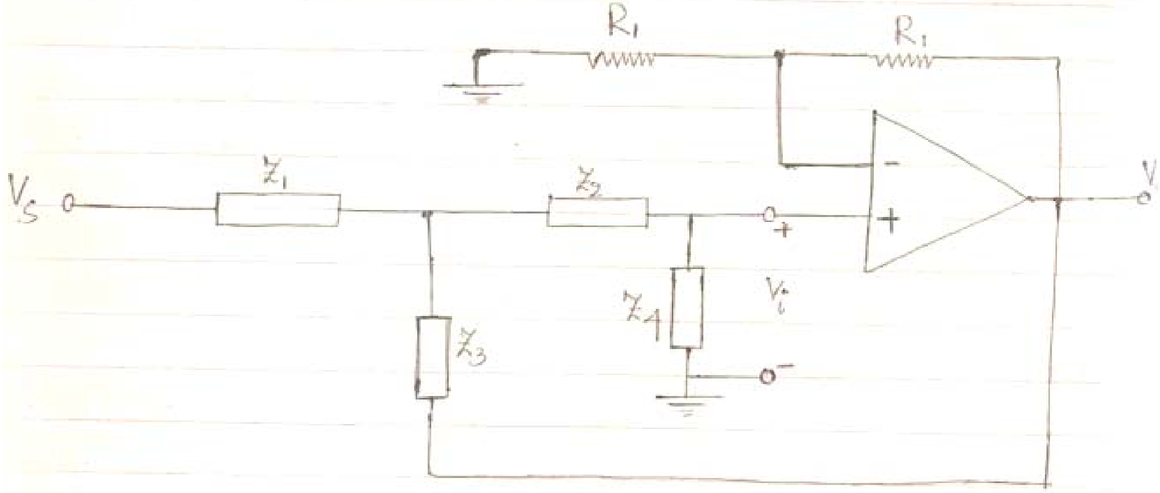


FIG : 4 (a) : Generalized active filter Prototype

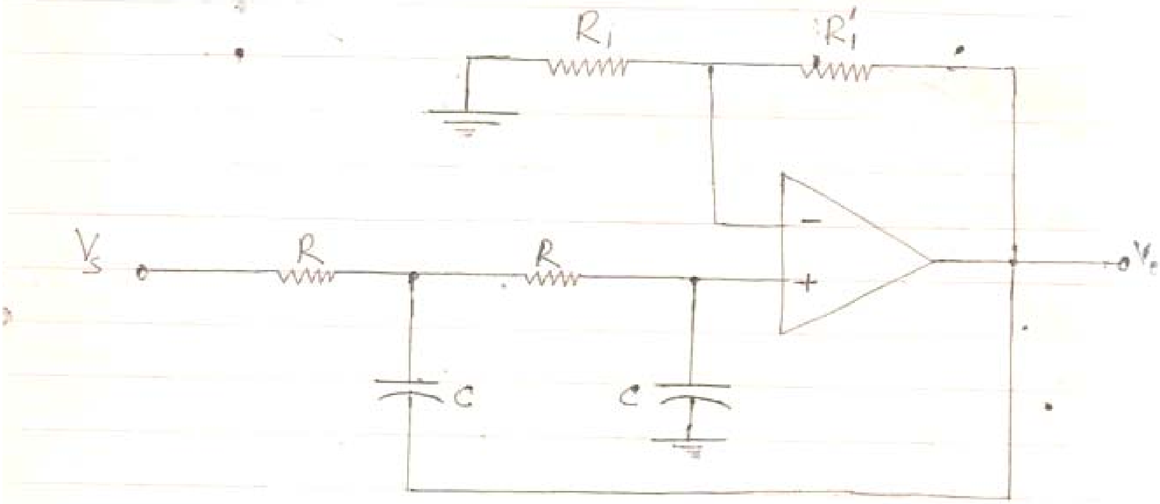
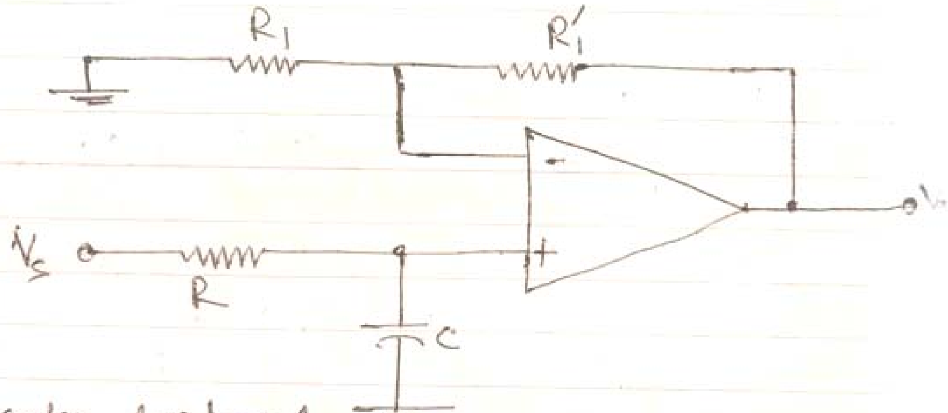


FIG : 4 (b). Second order Low pass Section.



4(c) First order low pass Section

In fig 4(b) the active element is the operational amplifier. Its stable mid-band gain is given by : $A_{V_0} = \frac{V_0}{V_i} = \frac{(R_1 + R_1')}{R_1}$ — (10).
Assuming the amplifier input current to be zero it can be shown that :

$$A_V(s) = \frac{V_0}{V_s} = \frac{A_{V_0} Z_3 Z_4}{Z_3(Z_1+Z_2+Z_3)+Z_1Z_2+Z_1Z_4(1-A_{V_0})} \quad \text{— (11)}$$

For this network to be low pass filter [FIG 4(b)].

$Z_1 = Z_2 = R$ (Resistances)

& $C_3 = C_4 = C$ (capacitances)

∴ The transfer function of this network takes the form :

$$A_V(s) = A_{V_0} \frac{(1/RC)^2}{\left[s^2 + \left(\frac{3-A_{V_0}}{RC}\right)s + \left(\frac{1}{RC}\right)^2\right]} \quad \text{— (12)}$$

Comparing eq^n (8) & (12) :

$$\omega_0 = \frac{1}{RC} \quad \text{— (13)}$$

& $2K = 3 - A_{V_0}$

or $A_{V_0} = 3 - 2K$ — (14)

Therefore by cascading prototype of the form 4(b) using identical R's and C's and selecting the gain A_{V_0} of each OP-AMP so that it satisfy eq^n (14) even order Butterworth filter can be synthesised.

Similarly to get odd order filters it is necessary to cascade the 1st order filter eq^n (9) with 2ND order section as in 4(b). The transfer function for 1st order is given by eq^n (9) for arbitrary A_{V_0} provided W_0 satisfy eq^n (13).

HIGH PASS PROTOTYPE: For the ideal high pass filter its characteristic is shown in fig 1(b). The high pass 2ND order filter can be obtained from low pass 2ND order prototype of eq^n (8) by applying the transformation

$$\left.\frac{S}{\omega_0}\right|_{\text{low pass}} \longrightarrow \left.\frac{W_0}{S}\right|_{\text{high pass}}$$

Thus by interchanging R's and C's of fig 4(b) high pass 2ND order active filter can be obtained.

Band Pass Filter: By Cascading a low pass 2ND order Section having cut-off frequency f_{OH} with a high pass 2ND order Section whose cutoff frequency is f_{OL}, providing $f_{OH} > f_{OL}$. We can get a 2ND order band pass prototype as shown in fig 1(C).

Band-Reject filter (Band Ellimination Filter): A band reject filter Can be obtained by the parallel connection of a high pass section whose cutoff frequency is f_{OL} with a low pass Section whose cutoff frequency is f_{OH}. For band Reject Charcecteristic it is required that $f_{OH} < f_{OL}$.

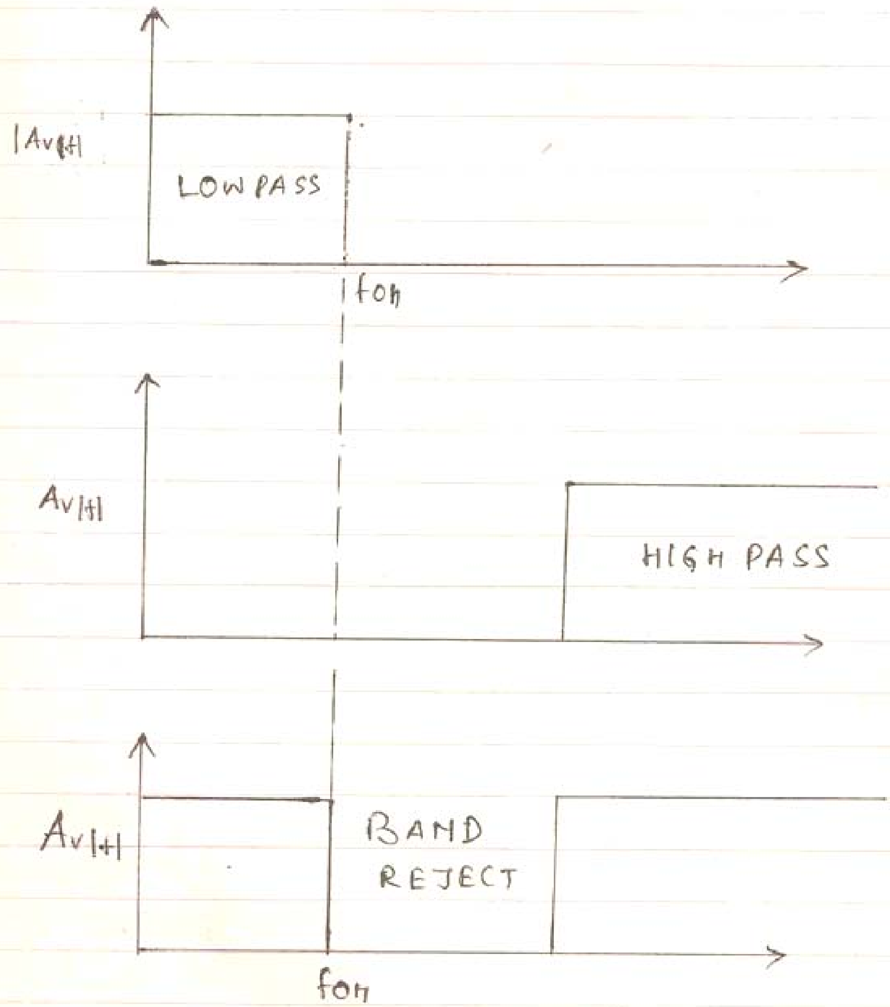


FIG : 5(A).