# 2025

1. Choose the correct answers to the questions from the given options.

   (Do not copy the questions, write the correct answers only.)

   (i) The given quadratic equation $3x^2 + \sqrt{7}x + 2 = 0$ has

   (a) Two equal real roots  (b) Two distinct real roots

   (c) More than two real roots  (d) No real roots

**Answer:**

To determine the nature of the roots of the quadratic equation:

$$3x^2 + \sqrt{7}x + 2 = 0$$

We use the **discriminant** of the quadratic equation $ax^2 + bx + c = 0$, which is:

$$D = b^2 - 4ac$$

Here:

- $a = 3$
- $b = \sqrt{7}$
- $c = 2$

Now calculate:

$$D = (\sqrt{7})^2 - 4(3)(2) = 7 - 24 = -17$$

Since the discriminant is **negative**, the quadratic equation has:

**(d) No real roots** ✅

2. (i) Solve the following quadratic equation $2x^2 - 5x - 4 = 0$ **4 Marks**
Give your answer correct to three significant figures.
(*Use mathematical tables for this question*)

**Answer:**

We are asked to solve the quadratic equation:

$$2x^2 - 5x - 4 = 0$$

This is in the standard form $ax^2 + bx + c = 0$, where:

- $a = 2$
- $b = -5$
- $c = -4$

## Step 1: Use the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-4)}}{2(2)} = \frac{5 \pm \sqrt{25 + 32}}{4} = \frac{5 \pm \sqrt{57}}{4}$$

Now we use mathematical tables or a calculator to find:

$$\sqrt{57} \approx 7.55 \quad \text{(to three significant figures)}$$

## Step 2: Calculate the Two Roots

$$x_1 = \frac{5 + 7.55}{4} = \frac{12.55}{4} \approx 3.14$$

$$x_2 = \frac{5 - 7.55}{4} = \frac{-2.55}{4} \approx -0.6375 \approx -0.638$$

## ✅ Final Answer (to 3 significant figures):

$$x = 3.14 \quad \text{or} \quad x = -0.638$$

**2024**

(iii) The roots of the quadratic equation $px^2 - qx + r = 0$ are real and equal if:

(a) $p^2 = 4qr$

(b) $q^2 = 4pr$

(c) $-q^2 = 4pr$

(d) $p^2 > 4qr$

**Answer:**

The given quadratic equation is:

$$px^2 - qx + r = 0$$

To determine when the roots are **real and equal**, we use the **discriminant** $D$, which for a quadratic equation $ax^2 + bx + c = 0$ is:

$$D = b^2 - 4ac$$

Here:

- $a = p$
- $b = -q$
- $c = r$

So the discriminant becomes:

$$D = (-q)^2 - 4pr = q^2 - 4pr$$

For **real and equal** roots, the discriminant must be zero:

$$q^2 - 4pr = 0 \Rightarrow q^2 = 4pr$$

✅ **Correct answer: (b) $q^2 = 4pr$**

**Question 6**

(i) Solve the following quadratic equation for $x$ and give your answer correct to three significant figures: $\mathbf{2x^2 - 10x + 5 = 0}$ [3]

**(Use mathematical tables if necessary)**

**Answer:**

We are given the quadratic equation:

$$2x^2 - 10x + 5 = 0$$

This is in the standard form $ax^2 + bx + c = 0$, where:

- $a = 2$
- $b = -10$
- $c = 5$

### Step 1: Use the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values:

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(5)}}{2(2)} = \frac{10 \pm \sqrt{100 - 40}}{4} = \frac{10 \pm \sqrt{60}}{4}$$

Now use a table or calculator for the square root:

$$\sqrt{60} \approx 7.75 \quad \text{(to 3 significant figures)}$$

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### Step 2: Find the Two Roots

$$x_1 = \frac{10 + 7.75}{4} = \frac{17.75}{4} \approx 4.44$$

$$x_2 = \frac{10 - 7.75}{4} = \frac{2.25}{4} \approx 0.563$$

## ✅ Final Answer (to three significant figures):

$$x = 4.44 \quad \text{or} \quad x = 0.563$$

**2023**

(v) If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then $p$ is equal to:

(a) 4

(b) 3

(c) 5

(d) 2

**Answer:**

We are given the quadratic equation:

$$x^2 - px + 3 = 0$$

and told that **3 is a root** of the equation. That means if we substitute $x = 3$ into the equation, it must satisfy it.

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### Step 1: Substitute $x = 3$

$$(3)^2 - p(3) + 3 = 0 \Rightarrow 9 - 3p + 3 = 0 \Rightarrow 12 - 3p = 0 \Rightarrow 3p = 12 \Rightarrow p = 4$$

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### ✅ Final Answer: (a) 4

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**Question 3**

(i) Solve the following quadratic equation: [4]

$x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place.

*(Use mathematical tables if necessary.)*

**Answer:**

We are given the quadratic equation:

$$x^2 + 4x - 8 = 0$$

This is in the form $ax^2 + bx + c = 0$, where:

- $a = 1$
- $b = 4$
- $c = -8$

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### Step 1: Use the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)} = \frac{-4 \pm \sqrt{16 + 32}}{2} = \frac{-4 \pm \sqrt{48}}{2}$$

Now use tables or a calculator:

$$\sqrt{48} \approx 6.9 \quad \text{(rounded to 1 decimal place)}$$

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### Step 2: Find the Two Roots

$$x_1 = \frac{-4 + 6.9}{2} = \frac{2.9}{2} = 1.45 \approx \mathbf{1.5}$$

$$x_2 = \frac{-4 - 6.9}{2} = \frac{-10.9}{2} = -5.45 \approx -\mathbf{5.5}$$

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### ✅ Final Answer (to one decimal place):

$$x = 1.5 \quad \text{or} \quad x = -5.5$$

**2022**

**6. Which of the following quadratic equations has 2 and 3 as its roots?** **[1]**

(a) $x^2 - 5x + 6 = 0$ (b) $x^2 + 5x + 6 = 0$

(c) $x^2 - 5x - 6 = 0$ (d) $x^2 + 5x - 6 = 0$

**Answer:**

We are asked to find the quadratic equation that has **2 and 3** as its roots.

## Step 1: Use the Fact That:

If a quadratic has roots $\alpha = 2$ and $\beta = 3$, then the equation is:

$$(x-2)(x-3) = 0$$

Now expand it:

$$x^2 - 5x + 6 = 0$$

## ✅ Correct Answer: (a) $x^2 - 5x + 6 = 0$

**2022**

16. One of the roots of the quadratic equation $x^2 - 8x + 5 = 0$ is 7.3166. The root of the equation correct to 4 significant figures is [1]

(a) 7.3166 (b) 7.317

(c) 7.316 (d) 7.32

**Answer:**

We are given that one of the roots of the quadratic equation:

$$x^2 - 8x + 5 = 0$$

is **7.3166**, and we are to round this value **to 4 significant figures**.

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## Step 1: Identify the first 4 significant digits:

7.3166 → The first 4 significant figures are **7.316**

Now check the **5th digit** (which is the next one to the right):

- The 5th digit is **6**, which is **greater than 5**, so we **round up** the 4th digit.

So:

$$7.3166 \approx \boxed{7.317} \quad \text{(to 4 significant figures)}$$

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## ✅ Correct Answer: (b) 7.317

**2020**

**Question 1**

(a) Solve the following Quadratic Equation: [3]

$x^2 - 7x + 3 = 0$

Give your answer correct to two decimal places.

**Answer:**

We are given the quadratic equation:

$$x^2 - 7x + 3 = 0$$

This is in the standard form $ax^2 + bx + c = 0$, where:

- $a = 1$
- $b = -7$
- $c = 3$

### Step 1: Use the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute values:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(3)}}{2(1)} = \frac{7 \pm \sqrt{49 - 12}}{2} = \frac{7 \pm \sqrt{37}}{2}$$

Now find:

$$\sqrt{37} \approx 6.08 \quad \text{(rounded to two decimal places)}$$

### Step 2: Calculate the Two Roots

$$x_1 = \frac{7 + 6.08}{2} = \frac{13.08}{2} = \mathbf{6.54}$$

$$x_2 = \frac{7 - 6.08}{2} = \frac{0.92}{2} = \mathbf{0.46}$$

### ✅ Final Answer (to two decimal places):

$$x = 6.54 \quad \text{or} \quad x = 0.46$$

(b) Using properties of proportion find $x : y$, given:

$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

**Answer:**

We are given:

$$\frac{x^2 + 2x}{2x + 4} = \frac{y^2 + 3y}{3y + 9}$$

We are to find the ratio $x : y$ using properties of proportion.

### Step 1: Factor numerator and denominator

**Left-hand side:**

Numerator:

$$x^2 + 2x = x(x + 2)$$

Denominator:

$$2x + 4 = 2(x + 2)$$

So LHS becomes:

$$\frac{x(x+2)}{2(x+2)} = \frac{x}{2}$$

**Right-hand side:**

Numerator:

$$y^2 + 3y = y(y + 3)$$

Denominator:

$$3y + 9 = 3(y + 3)$$

So RHS becomes:

$$\frac{y(y+3)}{3(y+3)} = \frac{y}{3}$$

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### Step 2: Equating both sides

$$\frac{x}{2} = \frac{y}{3}$$

Cross-multiply:

$$3x = 2y \Rightarrow \frac{x}{y} = \frac{2}{3}$$

✅ **Final Answer:**

$$\boxed{x : y = 2 : 3}$$

(b) If $x = \dfrac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$, prove that $x^2 - 4ax + 1 = 0$ [4]

**Answer:**

We are given:

$$x = \frac{\sqrt{2a+1}+\sqrt{2a-1}}{\sqrt{2a+1}-\sqrt{2a-1}}$$

We are to **prove** that:

$$x^2 - 4ax + 1 = 0$$

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### ✅ Step 1: Let's denote:

Let

- $A = \sqrt{2a+1}$
- $B = \sqrt{2a-1}$

So the expression becomes:

$$x = \frac{A+B}{A-B}$$

### ✅ Step 2: Rationalize the expression

Multiply numerator and denominator by $A+B$:

$$x = \frac{(A+B)^2}{(A-B)(A+B)} = \frac{A^2+2AB+B^2}{A^2-B^2}$$

Now compute each:

- $A^2 = 2a+1$
- $B^2 = 2a-1$

So:

- Numerator = $A^2 + B^2 + 2AB = (2a+1) + (2a-1) + 2AB = 4a + 2AB$
- Denominator = $A^2 - B^2 = (2a+1) - (2a-1) = 2$

Therefore:

$$x = \frac{4a+2AB}{2} = 2a + AB$$

✅ **Step 3: Square both sides to find $x^2$**

We now have:

$$x = 2a + AB \Rightarrow x^2 = (2a + AB)^2 = 4a^2 + 4a \cdot AB + (AB)^2$$

Now compute $AB$:

$$AB = \sqrt{(2a+1)(2a-1)} = \sqrt{4a^2 - 1} \Rightarrow (AB)^2 = 4a^2 - 1$$

So:

$$x^2 = 4a^2 + 4a \cdot \sqrt{4a^2 - 1} + (4a^2 - 1) = 8a^2 - 1 + 4a\sqrt{4a^2 - 1}$$

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✅ **Step 4: Compute $4ax$**

$$4ax = 4a(2a + \sqrt{4a^2 - 1}) = 8a^2 + 4a\sqrt{4a^2 - 1}$$

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✅ **Step 5: Now compute $x^2 - 4ax + 1$**

$$x^2 - 4ax + 1 = [8a^2 - 1 + 4a\sqrt{4a^2 - 1}] - [8a^2 + 4a\sqrt{4a^2 - 1}] + 1$$

Simplify:

$$= (8a^2 - 1 + 4a\sqrt{4a^2 - 1}) - 8a^2 - 4a\sqrt{4a^2 - 1} + 1 = 0$$

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✅ **Final Conclusion:**

$$\boxed{x^2 - 4ax + 1 = 0}$$

✔ **Proved as required.**

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**2019**

(b) Solve for $x$ the quadratic equation $x^2 - 4x - 8 = 0$. [3]

Give your answer correct to three significant figures.

**Answer:**

Given:

$$x = \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}}$$

We need to **prove** that:

$$x^2 - 4ax + 1 = 0$$

### Step 1: Simplify $x$ by rationalizing the denominator

Multiply numerator and denominator by the conjugate of the denominator:

$$x = \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} - \sqrt{2a-1}} \times \frac{\sqrt{2a+1} + \sqrt{2a-1}}{\sqrt{2a+1} + \sqrt{2a-1}}$$

This gives:

$$x = \frac{(\sqrt{2a+1} + \sqrt{2a-1})^2}{(\sqrt{2a+1})^2 - (\sqrt{2a-1})^2}$$

### Step 2: Simplify numerator and denominator

Numerator:

$$(\sqrt{2a+1} + \sqrt{2a-1})^2 = (2a+1) + (2a-1) + 2\sqrt{(2a+1)(2a-1)} = 4a + 2\sqrt{4a^2 - 1}$$

Denominator:

$$(2a+1) - (2a-1) = 2$$

So:

$$x = \frac{4a + 2\sqrt{4a^2 - 1}}{2} = 2a + \sqrt{4a^2 - 1}$$

### Step 3: Square $x$

$$x^2 = (2a + \sqrt{4a^2 - 1})^2 = (2a)^2 + 2 \times 2a \times \sqrt{4a^2 - 1} + (\sqrt{4a^2 - 1})^2$$

$$= 4a^2 + 4a\sqrt{4a^2 - 1} + (4a^2 - 1)$$

$$= 8a^2 - 1 + 4a\sqrt{4a^2 - 1}$$

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### Step 4: Compute $4ax$

$$4ax = 4a(2a + \sqrt{4a^2 - 1}) = 8a^2 + 4a\sqrt{4a^2 - 1}$$

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### Step 5: Calculate $x^2 - 4ax + 1$

$$x^2 - 4ax + 1 = (8a^2 - 1 + 4a\sqrt{4a^2 - 1}) - (8a^2 + 4a\sqrt{4a^2 - 1}) + 1$$

Simplify:

$= 8a^2 - 1 + 4a\sqrt{4a^2 - 1}$ ↓ $a^2 - 4a\sqrt{4a^2 - 1} + 1 = 0$

## ✅ Conclusion:

$$x^2 - 4ax + 1 = 0$$

which proves the statement as required.

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**2018**

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. [4]

**Answer:**

Given the quadratic equation:

$$x^2 + 7x = 7$$

**Step 1: Rearrange into standard form**

$$x^2 + 7x - 7 = 0$$

**Step 2: Use the quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here:

- $a = 1$
- $b = 7$
- $c = -7$

Substitute values:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-7)}}{2(1)} = \frac{-7 \pm \sqrt{49 + 28}}{2} = \frac{-7 \pm \sqrt{77}}{2}$$

**Step 3: Calculate $\sqrt{77}$**

$$\sqrt{77} \approx 8.77$$

### Step 4: Find the two roots

$$x_1 = \frac{-7 + 8.77}{2} = \frac{1.77}{2} = 0.89$$

$$x_2 = \frac{-7 - 8.77}{2} = \frac{-15.77}{2} = -7.89$$

### ✅ Final Answer (to two decimal places):

$$x = 0.89 \quad \text{or} \quad x = -7.89$$